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# Editorial for Volume 9 Number 3: Models, Resources and Activities of Project Scheduling Problems

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Abstract—The Project Scheduling Problem (PSP) is a generic name given to a whole class of problems in which the best form, time, resources and costs for project scheduling are necessary. The PSP is an application area related to the project management. This paper aims at being a guide to understand PSP by presenting a survey of the general parameters of PSP: the Resources (those elements that realize the activities of a project), and the Activities (set of operations or own tasks of a person or organization); the mathematical models of the main variants of PSP and the algorithms used to solve the variants of the PSP. The project scheduling is an important task in project management. This paper contains mathematical models, resources, activities, and algorithms of project scheduling problems. The project scheduling problem has attracted researchers of the automotive industry, steel manufacturer, medical research, pharmaceutical research, telecommunication, industry, aviation industry, development of the software, manufacturing management, innovation and technology management, construction industry, government project management, financial services, machine scheduling, transportation management, and others. The project managers need to finish a project with the minimum cost and the maximum quality.

Keywords— Project Management; Manufacturing Management, Technology Management.

## 1. INTRODUCTION

The Project management is considered as The planning, monitoring and control of all aspects of a project and the motivation of all those involved in it to achieve the project objectives on time and to the specified cost, quality and performance [1]. A project is a temporary and unique effort that, with a set of resources, it looks forward to satisfying specific objectives in a period with certain time [2]. The application areas of the project management usually are defined in terms of: technical elements (development of software, pharmaceutical drugs or civil engineering, Production systems planning), elements of the administration (Project Scheduling Problems, Manufacturing Management [3], Technology Management, contracts with the government or development of new products), and groups of industry (industrial engineering, automobiles, chemicals or financial services).

This paper aims at being a guide to understand the Project Scheduling Problems (PSP) by presenting a survey of the general parameters of the PSP, the algorithms used to solve the problems and the differences of the variants of the problems. Section 2 presents the definition, classification and the general parameters of the PSP, Section 3 the variants of the PSP, Section 4 the algorithms used to solve the PSP, and the last section presents the conclusions.

## 2. PROJECT SCHEDULING PROBLEM

Kalelkar and Critical Path Method (CPM) is a project modelling technique and it is commonly used with all forms of projects, including construction, aerospace and defense, software development, research projects, product development, engineering, and plant maintenance, among others [4]. Any project with interdependent activities can apply this method of mathematical analysis. Although the original CPM program and approach is no longer used, the term is generally applied to any approach used to analyse a project network logic diagram. The concept of CPM known (at the moment) as Project Scheduling Problem.

The Project Scheduling Problem (PSP) is a generic name given to a whole class of problems in which the best form, time, resources and costs for project scheduling are necessary. The PSP is classified according to the optimization objective by which they were created, for example, PSP of general way looks to optimize: time, project cost and resource usage.

The general parameters of PSP are:

1. The resources are those elements that realize the activities of a project. Examples of discrete resources are: machines, tools, workers; continuous resources include: energy, liquids and money. The diverse types of resources are:

- Renewable, nonrenewable and doubly constrained resources (Slowinski, 1980; Weglarz, 1980). The renewable resources are represented by labour, machinery, equipment; the nonrenewable resources usually are represented by money; a doubly constrained resource can be either consumed (money, raw materials) or used (blades, cartridges) by activity during its execution.
- Preemptable or non-preemptable [5,6]. The preemptable resources are those resources used for the processing of current activity, allotted to another activity, and then returned to the previously interrupted activity whose processing may be resumed; the resources without this property are called non-preemptable resources.
- Reusable resources [7]. The reusable resources represent resources which act like renewable (recycling) but are consumed by little in each period they are used. Examples of reusable resources are tools with single or multiple cutting edges. The recycling takes time, and in consequence, each reusable resource becomes unavailable for some time periods.
- Dedicated resources [8]. The dedicated resources can be assigned to only one activity at a time.
- Spatial resources [9]. The spatial resources are resources used by a group of activities from the start of the first activity till the completion of the last activity of the group. Examples of spatial resources include dry docks of a shipyard, shop floor space, rooms, pallets, containers.
- Partially (non) renewable resources [10]. The partially (non) renewable resources are resources with availabilities (subsets of periods or time intervals).

- Unary resources (Baptiste et al., 1999). The unary resources are a special case of renewable resources with availabilities limited to one unit per period.

- Complementary resources [11]. The complementary resources are used for setting up resources and not used for the processing of activities. An example of a complementary resource is a set of workers who are capable of and responsible for properly setting up other resources like specialized machines, computers, or robots.

- Cumulative resources [12]. The concept of cumulative resources is a generalization of the concept of nonrenewable resources. The cumulative resources are considered when the availability of a resource in a given time period cumulatively depends on the utilization (depletions and replenishments) of this resource during the previous time periods. Examples of such resources are investment capital, storage facilities and intermediate products.

- Multi-skill resources [13]. The multi-skill resources are able (flexibility) to be allocated for different kinds of resource requirements. Each resource usually has more than one skill, it can still be assigned to only one activity at a time. Examples are staff members (each of them has his own set of skills), multi-skill machines, tools, robots. A multi-skill resource has the functionality of several renewable resources.

- Synchronizing resources [14]. The Synchronizing resources have the ability to ensure a simultaneous start of a set of activities to which these resources are allocated. Synchronizing resources are a special category of renewable resources.

- Allocating resources [14]. The allocatable resources can be viewed as renewable resources which require some setup operations before they are ready to process a given activity. If an activity $A_i$ requires a number of units of such a resource, they have to be allocated to this activity by another activity $A_k$ (so-called allocating activity) starting no later than activity $A_i$ .In other words, activity $A_k$ prepares a given number of units of an allocatable resource for processing activity $A_i$ . For example, a resource is equipment that has to be installed each time before it is used for executing some activity $A_i$. If this setup operation also requires some scarce renewable resources (like staff or tools), it can be modelled as an activity $A_k$.

- Adjacent resources [16,17]. The adjacent resources are resources with a physical location of a particular resource among resource (of the same type) are given, and it is important for activity processing. Examples of adjacent resources include adjacent parts of dry docks in a shipyard and check-in desks at airports, as well as other types of resources like processors in multi-processor or grid environments.

- Changeover resources [18]. The changeover resources are renewable resources with a setup (sequence-dependent) or must be changed over when passing from one activity $A_i$ to another activity $A_j$. For example: a project with several geographically distributed locations and some renewable resources can be used in each location, but in order to be put into service for processing activities in a given location, it is necessary to transport units of such resources from another location. The time needed for transport is treated as a setup (changeover) time.

- Auxiliary resources [19]. This is a similar concept of complementary resources of Artigues and Roubellat (2001).
- Heterogeneous resources [20]. The heterogeneous resources are renewable resources and multi-skills resources. An individual of a heterogeneous resource has the same skill, but some individuals may be less or more skilled than the others. The time needed to perform an activity and the quality of the performed work depend on the skill level of the individuals assigned to the activity.

2. The activities are defined as a set of operations or own tasks of a person or organization. The activities are characterized by resource requirements, activity processing model, and precedence constraints with other activities. The parameters related to the activities [21] are:

    - The resource requirement (resource demand, resource request) is the amount of a resource needed to execute a given activity.
    - Activity processing model describes a processing characteristic of an activity as a function of resource amounts allotted for this activity.
    - Time Parameters. The duration of a project activity can be determined by the time parameters: a ready time, a due date or a deadline. Ready time (also known as a release date) is a time at which an activity is ready for processing. In many projects scheduling problems ready times for all activities are identical to zero. Due date specifies a time limit for completing an activity. Usually, penalty costs per unit are considered for activities completed either before/after their due dates. The deadline can be defined similarly to the due date. The main difference is that a deadline cannot be violated.
    - Weight parameters. A weight is a cost / reward for executing an activity or a priority of an activity.
    - Setup time. The setups are operations performed in order to prepare resources for executing some activities. A setup is measured by the time needed to perform it (called setup time), and additionally by the cost of such an operation. The two types of setup times are: sequence-independent and sequence-dependent. A sequence-dependent setup time varies depending on the sequence of activities performed on a given resource unit, a sequence-independent setup time is the same for all possible sequences of activities.
    - Preemptibility. Each of the project activities may be either preemptable or non-preemptable.

## 3. VARIANTS OF THE PROJECT SCHEDULING PROBLEMS

The main variants of PSP are: Resource-constrained Project Scheduling Problem (RCPSP), Multi-Mode Resource-Constrained Project Scheduling Problem (MRCPSP), Construction Project Scheduling Problem (construction PSP), Project Scheduling Problem for Software Development (software PSP), Payment Scheduling Problem, and Time/cost trade-off Problem (TCTP).

1. Resource-constrained Project Scheduling Problem or RCPSP [22]. RCPSP is a problem of allocation of schedules that consists of scheduling a series of activities at a time determined with

limited resources, the activities are contemplated as renewable and nonrenewable resources that they are assigned in the course of the project [5]. RCPSP belongs to the class of the NP-hard problems [5]. The mathematical model of RCPSP [23] is formed by the equations 1-4: The equation 1 contains the objective function that minimizes the project duration (makespan) determined by the completion time (start time or S, since $d_{n+1} = 0$) of the dummy end activity $n + 1$. Constraints (equation 2) enforce the precedence constraints between activities. Constraints (equation 3) ensure for each resource $k$ and each time period $t$, that the resource demand of the set of activities in process $P(t)$ at time $t$ does not exceed the capacity, $T$ being an upper bound on the project's makespan. Where: $S$ is the start time, $n$ is the number of activities in the project, $n + 1$ is the unique dummy end activity, $d_i$ is the duration of activity $j$, $i$ and $j$ are A activities, $k$ are the resources, $K$ is the number of resources types, $t$ is a time period, $r$ are the units of resource, $R$ is the resource demand of a set of activities, $P(t)$ is a process at time $t$, $T$ is an upper bound on the project duration.

$$Min\ S_{n+1} \tag{1}$$

$$\text{subject to: } S_j - S_i \geq d_i, \quad (i, j) \in A \tag{2}$$

$$\sum_{P(t)} r_{ik} \leq R_k, \quad t = 1, \ldots, T;\ k = 1, \ldots, K \tag{3}$$

$$S_i \geq 0 \quad \forall i \tag{4}$$

2. Multi-Mode Resource-Constrained Project Scheduling Problem or MRCPSP [24]. The MRCPSP involves the selection of a time/resource combination for each activity such that the total project makespan is minimized. The MRCPSP is an extension of the well-known single-mode RCPSP. The problem of finding a feasible solution for the MRCPSP is NP-Complete problem [25]. The mathematical model of MRCPSP [24] is formed by the equations 5-10: Constraints (equation 6) ensure that each non-preemptable activity is performed exactly once in exactly one mode. Precedence constraints are guaranteed by (equation 7). Constraints (equation 8 and 9) ensure that the renewable and nonrenewable resource limits are not exceeded, respectively. Finally, the equation 10 is the binary status of the decision variables. Where: $R_k^P\left(R_k^v\right)$ is the number of available units of the *kth* (*lth*) renewable (nonrenewable) resource, $r_{jmk}^P$ is the number of units of the *kth* renewable resource ($k$ =1, …, $R$) required by activity $A_j$ executed in mode $m \in M_j$, $r$ is the number of units of the *lth* nonrenewable resource ($l$ = 1, …, $N$) required by activity $A_j$ executed in mode $m \in M_j$, $d_{jm}$ is the duration of activity $A_j$ executed in mode $m \in M_j$, $x_{jmt}$ =1 if activity $A_j$ executed in mode $m \in M_j$ is completed at the end of time period $t$ , $EF_j$, $LF_j$ are calculated assuming that the shortest duration mode is assigned to each activity, and the planning horizon $H$ is calculated for the modes with the longest durations.

$$\text{Minimize } \sum_{t=EF_{n+1}}^{LF_{n+1}} t x_{n+1,m,t} \tag{5}$$

$$\text{subject to } \sum_{m=1}^{|M_j|} \sum_{t=EF_j}^{LF_j} x_{jmt} = 1 \text{ for } j = 0, \ldots, n+1 \tag{6}$$

$$\sum_{m=1}^{|M_j|}\sum_{t=EF_i}^{LF_j} tx_{imt} \leq \sum_{m=1}^{|M_j|}\sum_{t=EF_j}^{LF_j} tx_{jmt} - d_{jm} \text{ for all } \left(A_i, A_j\right) \in P \tag{7}$$

$$\sum_{j=1}^{n}\sum_{m=1}^{|M_j|}\sum_{q=\max\{t,EF_j\}}^{\min\{t+d_{jm}-1,LF_j\}} r_{jmk}^{P} x_{jmq} \leq R_k^P \text{ for } k=1,\ ...,\ R;\ t=1,\ ...,\ H \tag{8}$$

$$\sum_{j=1}^{n}\sum_{m=1}^{|M_j|}\sum_{t=EF_j}^{LF_j} r_{j\ ml}^{v} \leq R_k^P \text{ for } l=1,\ ...,\ N \tag{9}$$

$$x_{jmt} \in \{0,1\} \text{ for } i=0,\ ...,\ n+1;\ m \in M_j;\ t=EF_j,\ ...,\ LF_j \tag{10}$$

3. Construction Project Scheduling Problem, construction PSP or CPSP [26]. The construction PSP consists of to select resources (workforce, machines) and their works to construction projects (for example scheduling of construction projects of highway construction scheduling), take into account particular conditions (technological and organizational methods, constraints resource availability to develop, and the optimal project schedule) where optimal means the accepted evaluation criteria (project duration, cost, quality) are met. The mathematical model of construction PSP [27] is formed by the equations 11-16: In the equation (11) is the objective function which minimizes the project completion time defined by minimizing the finish time of the unique dummy end activity $N + 1$. Constraints (12) ensure that no activity can be started until all its predecessors have been completed, and the dummy start activity 1 is assigned a value of 0. $d_j$ in constrained conditions 12, it is any selected from the set of satisfying the conditions (13) above. During any time interval (t - 1; t], thereinto the constraints (14) represent that the resource utilization does not exceed the resource availability levels for any of the deterministic resource types, constraint set (15) and (16) denote that chance-constrained of resource utilization do not exceed the resource availability level for any of the fuzzy and random resource types respectively. Where: *f* is the finish time, $N$+1 is the unique dummy end activity, $N$ is the number of activities in the project, $d_j$ is the duration of activity $j$ ($j \in$ UD), $f_j$ is the completion time (duration) of activity $j$, $H$ is the set of pairs of activities indicating finish-start precedent relation, $\beta_j$ is the probability or possibility for pre-given, $i$ is an activity, $k$ are the resources, $K$ is the number of resources types, $t$ is a time period, $r$ are the units of resource, $r_{ikt}$ is the amount of resource type $k$ that is required by activity $i$ at time $t$, $R$ is the resource demand of a set of activities, $R_{kt}$ is the total availability of deterministic resource type $k$ at time $t$, $a_{kt}$ is the total availability of fuzzy resource type $k$ at time $t$, $b_{kt}(\xi)$ is the total availability of random resource type $k$ at time $t$, ch, *Pos* is the possibility of fuzzy events, *Pr* is the probability of random events, *SR* is the set of total available resource requirements at the disposal of the project management is a random variable(process) with pre-given density function throughout the scheduling horizon, *FR* is the set of total available resource requirements at the disposal of the project management is a fuzzy number(be related to time) with pre-given membership function throughout the scheduling horizon, DR is the set of the total available resource requirements at the disposal of the project management is pre-given and fixed (for example $R_{kt}$) at time $t$, *UD* is the set of activity with random duration $UD_1$ or fuzzy duration $UD_2$ such that $UD = UD_1 + UD_2$.

$$\textit{Minimize } f_{N+1} \tag{11}$$

$$f_j - d_j \geq f_i \text{ and } f_1 = 0, \forall (i,j) \in H \tag{12}$$

$$Ch\left\{ d_j^{'} \leq d_j \leq d_j^{*} \right\} \geq \beta_j, j \in UD \tag{13}$$

$$\sum_{i \in S_t} r_{ikt} \leq R_{kt}, k \in DR \tag{14}$$

$$Pos\left\{ \sum_{i \in S_t} r_{ikt} \leq a_{kt} \right\} \geq \alpha_{kt}, k \in FR \tag{15}$$

$$\Pr\left\{ \sum_{i \in S_t} r_{ikt} \leq b_{kt}(\xi) \right\} \geq p_{kt}, k \in SR, \text{ for } t = 1, 2, \ ..., f_N \tag{16}$$

4. Project Scheduling Problem for Software Development, software PSP or PSPSD [28]. The software PSP consists of human resource allocation to the various tasks in a software development project in accordance with their skills to produce higher quality software, while keeping effort expenditure and schedule time to a minimum [29]. The mathematical model of software PSP [29] is formed by the equations 17-21: the equation (17) is the objective function of the software PSP, the equation (18) contains aspects related to project development cost, the equation (19) contains aspects related to project development duration, the equation (20) contains aspects related to task precedence relations of two consecutive task of the software development project, and the equation (21) contains aspects related to personnel availability. Where: $Pc$ is the cost per unit time of the $j^{th}$ personnel, $m$ is the total number of personnel allotted to the project development, $f_1$ is the total development cost, $P_{Tj}$ is the processing time of the $j^{th}$ task in the project, $Q_{Tj}$ is the queuing time of the $j^{th}$ task in the project for resource availability, $f_2$ is the actual duration of the project, $S_{Tj}$ is the starting time of the $j^{th}$ task, $F_{Tj}$ is the finish time of the $j^{th}$ task.

$$\min imize\ f(x) = f_1 + f_2 \tag{17}$$

$$f_1 = \sum_{j=1}^{m} Pc_j \tag{18}$$

$$f_2 = \sum_{j=1}^{m} \left( Q_{Tj} + P_{Tj} \right) \tag{19}$$

$$F_{T(j-1)} < S_{Tj} \quad \forall j \tag{20}$$

$$P_j(t) \Rightarrow \tau_i(t) \neq P_j(t) \Rightarrow \tau_k(t) \quad \forall i, j, k \tag{21}$$

5. Payment Scheduling Problem [30,31]. The Payment Scheduling Problem PSP consists of are several issues associated with analyzing trade-off in contract prices, profit margins, project deadlines and other payment parameters by developing models and solution methods for simultaneously determining the among, timing and location of payments in projects. The mathematical model of payment scheduling problem [32] is formed by the equations 22-24: The equation (22) is the objective function represents cash outflows, cash inflow and capital cost, where each component is discounted back to the beginning of the project, the equation (23) are the activity precedent constraints, the equation (24) are the capital constraints for each period of the project. It assumes that capital is a renewable resource, where the initial capital available c0 is augmented or reduced by cash flows that occur throughout the project. Where: $c_0$ is the total capital available at the beginning of the project in period 0, $l_k$ is the capital investment required by activity $k$ ($k$ = 1, 2, …, $m$), $F_{i(k)}$ is the cash outflow at the beginning of activity $k$ at node $i$, where each activity is defined by nodes $i$ and $j$, $F_{j(k)}$ is the cash

inflow received upon completion of activity $k$ at node $j$, $d_k$ is the duration of activity $k$, where $k$ may not be preempted, $T_{i(k)}$ is the time at which node $i$ of activity $k$ is scheduled to occur, $Z_{a(t)}$ is the set of activities ($a$) that are scheduled to be active in period $t$, $Z_{p(t)}$ is the set of activities ($p$) completed prior to period $t$, $\alpha$ is an opportunity cost of capital.

$$\text{Maximize} \sum_{k=1}^{m} F_{i(k)} \exp\left(-\alpha T_{i(k)}\right) + F_{j(k)} \exp\left(-\alpha T_{j(k)}\right) - \left[ l_k \exp\left(-\alpha d_k\right) - l_k \right] \exp\left(-\alpha T_{i(k)}\right) \quad (22)$$

$$T_{j(k)} - T_{i(k)} \geq d_k .\ k = 1, 2, ..., m \quad (23)$$

$$\sum_{k \in z_{a(t)}} l_k \leq c_0 + \sum_{k \in Z_{p(t)}} \left( F_{j(k)} + F_{i(k)} \right) \quad (24)$$

6. Time/cost trade-off Problem or TCTP [33,34]. The TCTP consists [35] consists of an algorithm for efficiently shortening the duration of a project when it exceeds the predetermined limit. Suppose that we are given a project network which represents with a set of activities (and their precedence relationships) to be performed (individual activity in one of several ways), each activity with its unique time and cost requirements. Suppose that we are given a project network which represents with a set of activities (and their precedence relationships) to be performed (individual activity in one of several ways), each activity with its unique time and cost requirements. Different decisions as to how the various activities are performed lead to different time-cost realizations for the overall network. The objective of the time-cost tradeoff problem for a project network is to identify the sets of decisions that result in desirable time-cost realizations. The mathematical model of $P\_C|T$ [36] is formed by the equations 25-28: The equation (25) reflects the cost minimization objective, equation (26) ensures that exactly one alternative is chosen for each activity, equation (27) maintains the precedence relationships among the activities; and equation (28) guarantees that the project will complete by its due-date. Where: $i$ are the activities, $n$ are the nodes, $j$ is the alternative, $a(i)$ is an alternative of activity, $c_{ij}$ is the cost, $x_{ij}$ is a 0-1 variable which is 1 if alternative $j$ is selected for executing activity $i$ and 0 otherwise, $t_{ij}$ is the time, $s_i$ ($s_i \geq 0$) is the start time for activity $i$, $S(i)$ is the set of the immediate successors of $i$, $n$+1 is the finish node, $d$ is the due date.

$$\min \sum_{1 \leq i \leq n} \sum_{1 \leq j \leq a(i)} c_{ij} x_{ij} \quad (25)$$

$$\sum_{1 \leq j \leq a(i)} x_{ij} = 1 \text{ for all } i = 1, ..., n \quad (26)$$

$$\sum_{1 \leq j \leq a(i)} t_{ij} x_{ij} + s_i \leq s_k \text{ for all } k \in S(i) \quad (27)$$

$$s_{n+1} \leq d \quad (28)$$

## 4. ALGORITHMS TO SOLVE THE PROJECT SCHEDULING PROBLEMS

In the specialized literature there exist various approaches for carrying out the solution of the Project Scheduling Problems by means algorithms. Table 1 shows diverse algorithms to solve the Project Scheduling Problems and their variants. Where: 1. The Tabu search algorithm applied to the RCPSP and their variants were used by [37,38,39]. 2. The Simulated Annealing algorithm applied to the RCPSP and their variants were used by [40,41,42]. 3. The Genetic Algorithm applied to the RCPSP and their variants were used by [43, 44, 45, 46, 47, 48, 49, 50]. 4. Others algorithms (branch-and-bound) applied to the RCPSP and their variants were used by [51, 52, 53]. 5. The Tabu Search algorithm applied to the MRCPSP and their variants were used by [54, 55]. 6. The Simulated Annealing algorithm applied to the MRCPSP and their variants were used by [41, 57, 58, 59, 61,62]. 7. The Genetic Algorithm applied to the MRCPSP and their variants were used by [61, 63, 64, 65, 66, 67, 68]. 8. Others algorithms (branch-and-bound) applied to the MRCPSP and their variants were used by [51]. The Particle Swarm Optimization by [71, 72]. 9. The Simulated Annealing algorithm applied to the construction PSP and their variants were used by [73, 74]. 10. The Genetic Algorithm applied to the construction PSP and their variants were used by [74, 76]. 11. The Genetic Algorithm applied to the software PSP and their variants were used by [77, 78]. 12. Others algorithms (Particle Swarm Optimization) applied to the software PSP and their variants were used by [29]. 13. The Tabu search algorithm applied to the Payment Scheduling Problem and their variants was used by [79]. 14. The Simulated Annealing algorithm applied to the Payment Scheduling Problem and their variants was used by [61, 79]. 15. The Genetic Algorithm applied to the Payment Scheduling Problem and their variants was used by [80]. 16. Others algorithms (Hybrid Simulated Annealing and Genetic Algorithm) applied to the Payment Scheduling Problem and their variants were used by [80]. 17. The Tabu search algorithm applied to the TCTP and their variants were used by [81]. 18. Genetic Algorithm applied to the TCTP and their variants were used by [82, 83, 84]. 19. Others algorithms (branch-and-bound) applied to the TCTP and their variants were used by [85]. The hybrid scatter-search [86].

The RCPSP variants: Single-Mode Resource-Constrained Project Scheduling Problem [79], high school timetabling [87, 88], University Course Timetabling [87, 88], the audit-scheduling problem [89]. The MRCPSP variants: Multi-mode resource-constrained project scheduling problem with discounted cash flows or MRCPSPDCF [90, 91], multi-skill project scheduling problem or MSPSP [92, 93]. The construction PSP variants: larger-scale construction project scheduling problem with serious resource conflict [73], and Multi-Site Construction Problem Scheduling [95]. The software PSP variants: Fuzzy project scheduling system for software development [96], Time depend software Project Scheduling Problem [56], A project scheduling problem with labour constraints and time-dependent activities requirements [60], Project Scheduling Problem for Software Development with Random Fuzzy Activity Duration Times [28], and Fuzzy resource-constrained project scheduling problem for software development [75]. The Payment Scheduling Problem variants: Contractor’s payment scheduling problem [69], client´s payment scheduling problem [80], Joint payment scheduling Problem [94], max-NPV project scheduling problem [70], multi-mode project payment scheduling problem or MPPSP [62]. The TCTP variants: deadline problem [106], budget problem [36], time-cost tradeoff problem in fuzzy environment [15].

Table 1 algorithms to solve project scheduling problems and their variants.

| Algorithms / methods | RCPSP and variants | MRCPSP and variants | Construction PSP and variants | Software PSP and variants | Payment Scheduling Problem | TCTP and variants |
|---|---|---|---|---|---|---|
| Tabu Search | 1 | 5 | - | - | 13 | 17 |
| Simulated Annealing | 2 | 6 | 9 | - | 14 | - |
| Genetic Algorithm | 3 | 7 | 10 | 11 | 15 | 18 |
| Others | 4 | 8 | - | 12 | 16 | 19 |

## 5. CONCLUSIONS

This paper was a guide to understand Project Scheduling Problem (PSP) and their variants, contains a summary of the related works of the PSP, it provides a comprehensive view of the state of research and practice in the domain of PSP, diverse options appeared to solve PSP with costs, resources, time, and so on. We find in the related works that all the authors mention the term Project Scheduling Problem as the same that the term Resource-constrained Project Scheduling Problem, but we think necessary to make a difference with a new definition (section 2 of this paper) of the Project Scheduling Problem as a generic name given to a whole class of problems, including RCPSP and other problems. Finally, we conclude that, it´s necessary to create new (others) mathematical models of real problems related to PSP, and more algorithms, techniques and applications that could solve the real problems, for example, to solve the scheduling of innovation and technology projects and scheduling of government projects, and others. The future works of this research are a focus to try to solve these problems.

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## REFERENCES

[1] Atkuinson, R. (1999). Project management: cost, time and quality, two best guesses and a phenomenon, its time to accept other success criteria. International Journal of Project Management, 17(6): 337-342.

[2] Kimms, A. (2001). Mathematical Programming and Financial Objectives for Scheduling Projects, Publisher: Kluwer, Boston.

[3] Modrak, V., and Dima, I.C. (2013). Recent Directions in Production and Operation Management: A Survey. International Journal of Industrial Engineering, 20(9-10): 526-533.

[4] Kelly, J.E. (1963). The Critical-path Method: Resources Planning and Scheduling. Prentice-Hall, Englewood Cliffs, N.J

[5] Blazewicz, J., Lenstra, J.K. and Rinnooy Kan, A.H.G. (1983). Scheduling subject to resource constraints: Classification and complexity. Discrete Applied Mathematics, 5 (1): 11-24.

[6] Blazewicz, J., Drabowski, M. and Weglarz, J. (1986). Scheduling multiprocessor tasks to minimize schedule length. IEEE Transactions on Computers, 35:389-393

[7] Shewchuk, J.P. and Chang, T.C. (1995). Resource-constrained job scheduling with recyclable resources. European Journal of Operational Research, 81(2): 364-375.

[8] Bianco, L., Dell'Olmo, P. and Speranza, M.G. (1998). Heuristics for multi-mode scheduling problems with dedicated resources. European Journal of Operational Research, 107 (2): 260-271.

[9] De Boer, R. (1998). Resource-constrained multi-project management, a hierarchical decision support system. Ph.D. Thesis, University of Twente.

[10] Böttcher, J., Drexl, A., Kolisch, R. and Salewski, F. (1999). Project scheduling under partially renewable resource constraints. Management Science, 45 (4): 543-559.

[11] Artigues, C. and Roubellat, F. (2001). A Petri net model and a general method for on and off-line multi-resource job shop scheduling with setup times. International Journal of Production Economics, 74 (1-3): 63-75.

[12] Neumann, K., Schwindt, C. and Zimmermann, J. (2002). Project Scheduling with Time Windows and Scarce Resources: Temporal and Resource-Constrained Project Scheduling with Regular and Nonregular Objective Functions. Springer, Berlin

[13] Néron, E. (2002). Lower bounds for the multi-skill project scheduling problem. Proceedings of the Eighth International Workshop on Project Management and Scheduling (PMS2002), Valencia, Spain, 274-277.

[14] Schwindt, C. and Trautmann, N. (2003). Scheduling the production of rolling ingots: industrial context, model, and solution method. International Transactions in Operational Research, 10 (6): 547-563

[15] Ghazanfari, M., Shahanaghi, K. and Yousefli, A. (2008). An Application of Possibility Goal Programming to the Time-Cost Trade off Problem. Department of Industrial Engineering, Iran University of Science and Technology, Tehram, Iran. Journal of Uncertain Systems, 2 (1): 22-30.

[16] Duin, C.W. and Van Der Sluis, E. (2006). On the complexity of adjacent resource scheduling. Journal of Scheduling, 9 (1): 49-62.

[17] Paulus, J.J. and Hurink, J. (2006). Adjacent-resource scheduling: why spatial resources are so hard to incorporate. Electronic Notes in Discrete Mathematics, 25: 113-116.

[18] Neumann, K., Schwindt, C. and Zimmermann, J. (2006). Resource-constrained project scheduling with time windows. Józefowska, J., Weglarz, J. (Eds.), Perspectives in Modern Project Scheduling. Springer, Berlin, 375-407.

[19] Mika, M., Waligóra, G. and Weglarz, J. (2006). Modelling setup times in project scheduling. In: Józefowska, J., We̜glarz, J. (Eds.), Perspectives in Modern Project Scheduling. Springer, Berlin, 131-163.

[20] Tiwari, V., Patterson, J.H. and Mabert, V.A. (2009). Scheduling projects with heterogeneous resources to meet time and quality objectives. European Journal of Operational Research, 193(3): 780-790.

[21] Weglarz, J. (1980). On certain models of resources allocation problems. Kybernetes, 9(1), 61-66.

[22] Pritsker, A.A.B., Watters, L.J. and Wolfe, P.M. (1969). Multiproject scheduling with limited resources: A zero-one programming approach. Management Science, 16: 93-107.

[23] Tormos, P. and Lova, A. (2001). A Competitive Heuristic Solution Technique for Resource-Constrained Project Scheduling. Annals of Operations Research, 102: 65-81.

[24] Talbot, F.B. (1982). Resource-constrained project scheduling with time-resource trade-offs: the nonpreemptive case. Management Science, 28(10): 1197-1210.

[25] Kolisch, R. and Drexl, A. (1997). Local Search for Nonpreemptive Multi-Mode Resource-Constrained Project Scheduling. IIE Transactions, 29(11): 987-999.

[26] Adeli, H. and Karim, A. (1997). Scheduling/cost optimization and neural dynamics model for construction. Journal of Construction Engineering and Management, 123: 450-458.

[27] Julin, H. and Zhong-Ping, W. and Guochun, T. (2004). Construction project scheduling problem with uncertain resource constraints. Research Report School of Mathematics and Statistics, Wuhan University.

[28] Huang, W., Ding, L., Wen, B. and Cao, B. (2009). Project Scheduling Problem for Software Development with Random Fuzzy Activity Duration Times. Advances in Neural Networks – ISNN 2009, 5552: 60-69.

[29] Gonsalves, T. and Itoh, K. (2010). Multi-Objetive Optimization for Software Development Projects. International MultiConference of Engineers and Computer Scientists 2010, IMECS 2010, Vol. 1, March 17-19, Hong Kong.

[30] Russell, A.H. (1970). Cash Flows in Networks. Management Science, 6(5): 357-372.

[31] Grinold, R.C. (1972). The payment scheduling problem. Naval Research Logistics, 19.

[32] Doersch, E.W. and Patterson, J.H. (1977). Scheduling a project to maximize its present value: A zero-one programming approach. Management Science, 23 (8): 882-889.

[33] Kelley, J.E. (1961). Critical-Path Planning and Scheduling: Mathematical Basis. Operations Research, 9(3): 296-320.

[34] Fulkerson, D.R. (1961) A network flow computation for project cost curves, Management Science 7, 167-178

[35] Siemens, N. (1971). A simple CPM Time-Cost Tradeoff Algorithm. Management Science, 17(6): 354-363.

[36] De, P., James Dunne, E., Ghosh, J.B. and Wells, C.E. (1995). The discrete time-cost tradeoff problem revisited. European Journal of Operational Research, 81(2): 225-238

[37] Baar, T., Brucker, P. and Knust, S. (1998). Tabu-search algorithms and lower bounds for resource-constrained scheduling problem. In: Voss, S., Martello, S., Osman, I., Roucairol, C. (Eds.). Meta-Heuristics: Advances and Trends in Local Search. Paradigms for Optimization. Kluwer Academic, 1-18.

[38] Nonobe, K. and Ibaraki, T. (2002). Formulation and tabu search algorithm for the resource constrained project scheduling problem. Ribeiro, C.C., Hansen, P. (Eds.), Essays and Surveys in Metaheuristics. Kluwer Academic Publishers, 557-58

[39] Pinson, E., Prins, C. and Rullier, F. (1994). Using tabu search for solving the resource-constrained project scheduling problem. Proceedings of the Fourth International Workshop on Project Management and Scheduling, Leuven, Belgium, 102-106.

[40] Boctor, F.F. (1996). Resource-constrained project scheduling by simulated annealing. International Journal in Production Research, 34: 2335-2351.

[41] Bouleimen, K. and Lecocq, H. (2003). A new efficient simulated annealing algorithm for the resource-constrained project scheduling problem and its multiple mode version. European Journal of Operational Research, 149: 268-281.

[42] Cho, J.-H. and Kim, Y.-D. (1997). A simulated annealing algorithm for resource-constrained project scheduling problems. Journal of the Operational Research Society, 48: 735-744.

[43] Alcaraz, J.and Maroto, C. (2001). A robust genetic algorithm for resource allocation in project scheduling. Annals of Operations Research, 102: 83-109.

[44] Alcaraz, J., Maroto, C. and Ruiz, R. (2004). Improving the performance of genetic algorithms for RCPS problem. Proceedings of the Ninth International Workshop on Project Management and Scheduling, Nancy 2004, 40-43.

[45] Hartmann, S. (1998). A competitive genetic algorithm for resource-constrained project scheduling. Naval Research Logistics, 456: 733-750

[46] Kohlmorgen, U., Schmeck, H. and Haase, K. (1999). Experiences with fine-grained parallel genetic algorithms. Annals of Operations Research, 90: 203-219.

[47] Mendes, J.J.M., Goncalves, J.F. and Resende, M.G.C. (2009). A random key based genetic algorithm for the resource constrained project scheduling problem. Computers and Operations Research, 36(1): 92-109.

[48] Valls, V., Ballestin, F. and Quintanilla, M.S. (2003). A hybrid genetic algorithm for the RCPSP. Technical Report, Department of Statistics and Operations Research, University of Valencia.

[49] Valls, V., Ballestín, F. and Quintanilla, M.S. (2008). A hybrid genetic algorithm for the resource-constrained project scheduling problem. European Journal of Operational Research, 185: 495-50

[50] Hindi, K.S., Yang, H. and Fleszar, K. (2002). An evolutionary algorithm for resource-constrained project scheduling. IEEE Transactions on Evolutionary Computation, 6: 512-518.

[51] Demeulemeester, E. and Herroelen, W. (1992). A branch-and-bound procedure for multiple resource-constrained project scheduling problem. Management Science, 38(12): 1803-1818.

[52] Mingozi, A., Maniezzo, V., Ricciardelli, S. and Bianco, L. (1998). An exact algorithm for project scheduling with resource constraints based on a new mathematical formulation. Management Science, 44: 714-729.

[53] Patterson, J.H., Slowinski, R., Talbot, F.B. and Weglarz, J. (1989). Chapter 1: An algorithm for a general class of precedence and resource constrained scheduling problems. Sowinski, R., Weglarz, J. (Eds.), Advances in project scheduling. Elsevier, Amsterdam.

[54] Mika, M., Waligóra, G. and Węglarz, J. (2008). Tabu search for multi-mode resource-constrained project scheduling with schedule-dependent setup times. European Journal of Operational Research, 187(3): 1238-1250.

[55] Tchao, C. and Martins, S.L. (2008). Hybrid Heuristics for Multi-mode Resource-Constrained Project Scheduling. Learning and Intelligent Optimization. Lecture Notes in Computer Science, 5313:234-242

[56] Möhring, R.H., Schulz, A.S., Stork, F. and Uetz, M. (2002). Solving Project Scheduling Problems by Minimum cut computations. Working paper 4231-02. MIT Sloan School of Management.

[57] Slowinski, R. (1980). Two approaches to problems of resource allocation among project activities – a comparative study. Journal of the Operational Research Society, 31(8): 711-723.

[58] Boctor, F. (1996). An adaption of the simulated annealing for solving resource-constrained project scheduling problems. International Journal of Production Research, 34: 2335-2351.

[59] Jozefowska, J., Mika, M., Rozycki, R., Waligora, G. and Weglarz, J. (2001). Simulated annealing for multi-mode resource-constrained project scheduling. Annals of Operations Research, 102: 137-155.

[60] Drezet, L.E. and Billaut, J.C. (2008). A project scheduling problem with labor constraints and time-dependent activities requirements. International Journal of Production Economics, 112(1) 217-225.

[61] Chen, P-H. and Shahandashtia, S.M.(2009). Hybrid of genetic algorithm and simulated annealing for multiple project scheduling with multiple resource constraints. Automation in Construction, 18(4): 434-443.

[62] He, Z. and Xua, Y. (2008). Multi-mode project payment scheduling problem (MPPSP) with bonus-penalty structure. European Journal of Operational Research, 189(3): 1191-1207.

[63] Van Peteghem, V. and Vanhoucke, M. (2010). A genetic algorithm for the preemptive and non-preemptive multi-mode resource-constrained project scheduling problem. European Journal of Operational Research, 201(2): 409-418.

[64] Mori, M., Tseng, C. (1997). A genetic algorithm for multi-mode resource constrained project scheduling problem. European Journal of Operational Research, 100: 134-141.

[65] Özdamar, L. (1999). A genetic algorithm approach to a general category project scheduling problem. IEEE Transactions on Systems, Man, and Cybernetics, Part C: Applications and Reviews, 29: 44–59.

[66] Hartmann, S. (2001). Project scheduling with multiple modes: A genetic algorithm. Annals of Operations Research, 102: 111-135.

[67] Alcaraz, J., Maroto, C., and Ruiz, R. (2003). Solving the multi-mode resource-constrained project scheduling problem with genetic algorithms. Journal of the Operational Research Society, 54: 614–626.

[68] Lova, A., Tormos, P., Cervantes, M. and Barber, F. (2009). An efficient hybrid genetic algorithm for scheduling projects with resource constraints and multiple execution modes. International Journal of Production Economics, 117(2): 302-316.

[69] Dayanand, N. and Padman, R. (1993). Payments in projects: A contractor's model. Working Paper 93-71, The Heinz School, Carnegie Mellon University, Pittsburgh, PA.

[70] Herroelen, W.S., Dommelen, P. and Demeulemeester, E.L. (1997). Project network models with discounted cash flows: A guided tour through recent developments. European Journal of Operational Research, 100: 97-121

[71] Zhang, H., Tam, C. and Li, H. (2006). Multimode project scheduling based on particle swarm optimization. Computer-Aided Civil and Infrastructure Engineering, 21: 93-103.

[72] Jarboui, B., Damak, N., Siarry, P. and Rebai, A. (2008). A combinatorial particle swarm optimization for solving multi-mode resource-constrained project scheduling problems. Applied Mathematics and Computation, 195: 299-308.

[73] Liu, T., Liu, M., Zhang, Y.-B. and Zhang, L.(2005). Hybrid genetic algorithm based on synthetical level of resource conflict for complex construction project scheduling problem. International Conference on Machine Learning and Cybernetics, 9: 5699 - 5703.

[74] Leu, S-S., Chen, A-T. and Yang, C-H. (2001). A genetic algorithm-based fuzzy optimal model for construction. International Journal of Project Management , 19: 47-58.

[75] Wang, X. and Huang, W. (2010). Fuzzy resource-constrained project scheduling problem for software development. Wuhan University Journal of Natural Sciences, 15 (1): 25-30.

[76] Hegazy, T. (1999). Optimization of construction time–cost trade-off analysis: using genetic algorithms. Canadian Journal of Civil Engineering, 26(6):685-697.

[77] Chang, C.K., Jiang, H-y., Di, Y., Zhu, D. and Ge, Y.(2008). Time-line based model for software project scheduling with genetic algorithms. Information and Software Technology, 50(11): 1142-1154.

[78] Alba, E. and Chicano, J.F. (2007). Software project management with GAs. Information Sciences, 177(11):Pages 2380-2401.

[79] Alcaraz, J.and Maroto, C. (2001). A robust genetic algorithm for resource allocation in project scheduling. Annals of Operations Research, 102: 83-109.

[80] Dayanand, N. and Padman, R. (1995). Project contracts and payment schedules: the client's problem. Working Paper 95-23, The Heinz School, Carnegie Mellon University, Pittsburgh, PA.

[81] De Reyck, B., Demeulemeester, E. and Herroelen,W. (1998). Local search methods for the discrete time/resource trade-off problem in project networks. Naval Research Logistic Quarterly, 45: 553–578.

[82] Eshtehardian, E., Afsharb, A. and Abbasniac,R. (2009). Fuzzy-based MOGA approach to stochastic time–cost trade-off problem. Automation in Construction, 18(5): 692-701.

[83] Leu, S.S., Chen, A.T. and Yang, C.H. (2001). A GA-based fuzzy optimal model for construction time–cost trade-off, International Journal of Project Management, 19(1): 47-58.

[84] Ranjbar, M.R. and Kianfar, F. (2007). Solving the discrete time/resource trade-off problem in project scheduling with genetic algorithms.Applied Mathematics and Computation, 191(2): 451-456.

[85] Demeulemeester, E., De Reyck, B. and HerroelenW.(2000). The discrete time/resource trade-off problem in project networks: a branch and bound approach. IIE Transaction, 32(11):1059–1069.

[86] Ranjbar, M., De Reyck, B. and Kianfar, F. (2008). A hybrid scatter-search for the discrete time/resource trade-off problem in project scheduling. European Journal of Operational Research 193 (1), 35-48.

[87] Brucker, P. and Knust, S. (2001). Resource-constrained project scheduling and timetabling. In: Bruke, E., Erben, W. (Eds.), Practice and Theory of Automated Timetabling III (PATAT III). Lecture Notes in Computer Science, 2079: 277-293.

[88] Schaerf, A. (1999). Local search techniques for large high school timetabling problems. IEEE Transactions on Systems, Man and Cybernetics, Part A: systems and human, 29(4): 368-377.

[89] Brucker, P. and Schumacher, D. (1999). A new tabu search procedure for audit-scheduling problem. Journal of Scheduling, 2: 157-173.

[90] Brucker, P., Drexl, A., Mohring, R. and Neumann, K. (1999). Resource-constrained project scheduling: Notation, classification, models and methods. European Journal of Operational Research, 112: 3-41.

[91] Ulusoy, G. (2001). Four paymentmodels for themulti-mode resource constrained project scheduling problem with discounted cash flows. Annals of Operations Research, 102: 237-261.

[92] Bellenguez, O. and Néron, E. (2005). Lower bounds for the multi-skill project scheduling problem with hierarchical level of skills. In: Burke, E.K., Trick, M. (Eds.), Lecture Notes in Computer Science (3616): Practice and Theory of Automated Timetabling V. Springer, Berlin, 229-243.

[93] Bellenguez-Morineau, O. (2008). Methods to solve multi-skill project scheduling problem. 4OR: A Quarterly Journal of Operations Research, 6 (1): 85-88.

[94] Dayanand, N. and Padman, R. (1997). On modelling payments in projects. Journal of the Operational Research Society, 48: 906-918

[95] Le, H.Q. and Rüppel, U. (2009). Multi-site Construction Project Scheduling considering resource moving time in developing countries. 18th International Conference on the Application of Computer Science and Mathematics in Architecture and Civil Engineering. K. Gürlebeck and C. Könke (eds.), Weimar, Germany.

[96] Hapke, M., Jaszkiewicza, A. and Slowinskia, R. (1994). Fuzzy project scheduling system for software development. Fuzzy Sets and Systems, 67(1): 101-117.