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Capacitated Multi-Facility Location Problem on the Ellipsoid with Multi-Zone Restrictions

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Abstract. One of the main problems faced by organizations is the strategic location of its facilities. This is because resource acquisition and operational performance of the supply chain depend on this aspect. Complexity is increased if the most suitable location is not available due to zone restrictions. While facility location models have been developed to solve this problem, only a single zone restriction has been studied. The present work contributes to this context by (a) proposing a solving method for the multi-facility location problem with multi-zone restrictions, (b) considering the ellipsoidal surface of the Earth to provide more accurate estimates of distances, and (c) developing a large instance with real geographic location data for validation of the method and benchmark studies. The results reported in this work corroborate the suitability of the method and that, even with multi-zone restrictions, minimum distance/costs can be achieved when compared to the non-restricted problem.

Keywords: Facility Location Problem, Restricted Zones, Ellipsoidal Earth, Inverse Problem

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1. Introduction

In economic-administrative terms, the location of facilities is a factor that determines the feasibility of business and supply chain infrastructure. As discussed by Shih [1], ‘‘poor facility location decisions can lead to high transportation costs, inadequate supplies of raw materials and labour loss of competitive advantage, and financial loss [2]”. For this reason, it is very important to identify and evaluate the feasibility of locations based on investment and operational requirements.

Within the discipline of Operations Research, Location Theory is focused on the development of models to formally address facility location decisions. These models increase in complexity as more restrictions and variables are considered to properly represent real problems and thus, to provide adequate solutions [3].

Thus, facility location decisions involve several factors which are related to practical situations. Among the factors that affect facility location and re-location decisions the following can be mentioned [4,5]:

- Transportation infrastructure, means and costs
- Availability of labour force and salaries

- Location and availability of suppliers
- Market proximity
- Environmental and topography characteristics
- Waste disposal infrastructure
- Taxes and governmental regulations
- Availability of water, electric power and other supplies
- Social and cultural conditions (living conditions)
- Availability and reliability of support systems
- Irregular spatial distribution of customers

Another factor which is relevant to facility location problems is the surface model because distances and transportation routes depend on this aspect. Most works have considered the flat or spherical surface for facility location problems [6]. This assumption can lead to significant driving distance variations between widely separated location points [1]. The ellipsoidal surface model has led to more accurate estimations of distances between real geographical points.

The complexity of the problem is further increased when the best suitable location option, based on cost or distance, cannot be reached. In this aspect, few works on facility location have considered restrictions that define prohibited zones, congested areas, and barrier regions [7,8,9].

Hence, the present work extends on these aspects by developing a multi-facility location model on the ellipsoidal Earth with multi-zone restrictions. Also, a large test instance of geographical location points was developed to provide benchmark data for future research.

The present work is structured as follows: Section 2 presents the technical background of key aspects such as distances on the ellipsoid and zone restriction approaches. Then, Section 3 presents the development of the proposed multi-facility location model with multi-zone restrictions on the ellipsoidal Earth. The assessment of the model, including details of the solving method and the test instance, is presented and analyzed in Section 4. Finally, the conclusions and future work are presented in Section 5.

2. Technical Background

2.1 Ellipsoidal Model of the Earth

In Geoscience has developed theoretical studies referring to the size and shape of the Earth [10]. These studies have been very important in many contexts such as the optimization of aircraft routes and ships [11]. Thus, providing more accurate models of the Earth's size and shape have repercussions on costs, distances and/or times associated with the location of facilities [12].

In this context, the geoid, which is considered to represent the truer shape of the Earth, can be approximated as a reference ellipsoid [6,13]. As presented in Figure 1, the geodesic on an ellipsoid can be defined as the unique curve on the surface of the ellipsoid with the shortest distance between two points, where these points are determined by their latitude (λ) and longitude (ϕ) coordinates.

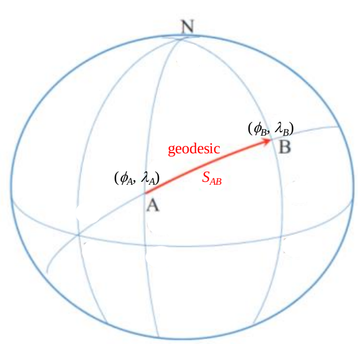

Figure 1. Geodesic on an ellipsoid[14].

When determining the length of the geodesic on the ellipsoid, it is important to mention the associated direct and inverse problems associated to geodesics [6,14,15]:

- ***Direct problem:*** given the latitude and longitude of a location A (λ_A, ϕ_A), the azimuth (direction) α_{AB}, and the geodetic distance S_{AB}, the problem consists of determining the latitude and longitude coordinates of location B (λ_B, ϕ_B) and the inverse azimuth.
- ***Inverse problem:*** given the latitudes and longitudes of two locations A and B (λ_A, ϕ_A, λ_B, ϕ_B), the problem consists of determining the azimuths and the geodetic distance S_{AB} between these locations.

Some solution methods have been proposed for both problems. Pittman [16] provided solutions through integrals while Deakin and Hunter [14] developed the Bessel method through elliptical integrals by series expansions. Kivioja [17] and Sjöberg et al. [18] provided strict solutions for the sphere and ellipsoidal correction through numerical integration.

For the purposes of this work the inverse problem is considered to estimate the geodesic distance (S_{AB}) between geographic locations. Particularly, the iterative method of Vincenty [15] was implemented due to its computational flexibility [6].

2.2 Zone Restrictions

The facility location problem seeks to determine the most suitable location for a facility (or set of facilities) to minimize the total cost or distance between it and a set of customers. When multiple facilities are considered, the minimization task depends also on determining which customers are to be assigned to each facility. This leads to define the multi-facility location problem as a NP-hard problem which is of high computational complexity [19].

In practice, minimization of distances or costs may not be the only factors to be considered by the facility location problem. Other variants of the problem, for example, when the possible locations are limited to a closed set, when there is a maximum distance restriction from a facility to the customers, or when the facility should not be located at the North of a specific line, adds complexity to the location task [20].

Given these variants, the importance of location models with zone restriction emerges, which may take different approaches:

- Restriction by region or area
- Restriction by flow or circulation
- Restriction by physical or geographic barrier
- Restriction by maximum distance

The work reported by Santra [21] focused on finding the locations of new facilities (multi-facility) considering a circular region around the center of gravity of a given number of existing facilities. The problem was addressed deterministically [21] and stochastically [22] on a plain surface with Euclidean distances. This work was extended to address the problem in a deterministic way with a triangular region on a plain surface with Euclidean distances [23].

Hamacher and Klamroth [24] presented theoretical and practical analyzes were presented. These were focused on locating a single facility considering a convex polyhedral barrier to restrict the crossing between facilities. This work also considered Euclidean distances and a plain surface.

Finally, other works [8,9] considered the problem of locating a new facility within a set of existing facilities and in the presence of a single region where the location of the facility and trips were not allowed. This region was defined as a convex polyhedral barrier on a plain surface with Euclidean distances.

These works provide reference to contrast the contribution of the present work which consists of the following:

- Plain surface and Euclidean distance have been considered in the reviewed works. Here, multiple facilities are to be located to minimize the total distance to customers on the ellipsoid, which is a more representative surface model of the Earth. Arc length on the ellipsoid is considered as the distance metric.

- Single restricted region has been considered in the reviewed works. Here, multiple circular restricted zones of different sizes are considered for the multi-facility location problem on the ellipsoid.

In the following section, the details of the proposed model are described.

3. Development of the Multi-Facility Location Model with Multi-Zone Restrictions on the Ellipsoid

In this work the continuous facility location problem of Weber is considered. This problem consists of finding the coordinate (x^*, y^*) of the facility that minimizes the sum of weighted distances between this point and n customer points with coordinates (a_i, b_i) where $i = 1, \ldots, n$. The Weber problem is continuous because (x^*, y^*) can take any value within the location space, and thus, its solution can lead to minimum coverage distance[19].
With these definitions and by considering the work of Chaves et al. [25], the objective function of the capacitated multi-facility Weber problem can be expressed as:

$$Minimize\ \sum_{i=1}^{n}\sum_{j=1}^{m} z_{ij} d_{ij} \tag{1}$$

Where $z_{ij} = 1$ if customer i is served by the facility j, and $z_{ij} = 0$ otherwise. d_{ij} = distance between the location of the facility at (x_j^*, y_j^*) and the assigned customer i at (a_i, b_i). Then, this function is subject to:

$$\sum_{j=1}^{m} z_{ij} = 1 \qquad \forall i \in n \tag{2}$$

$$\sum_{i=1}^{n} z_{ij} = n_j \qquad \forall j \in m \tag{3}$$

$$\sum_{i=1}^{n} p_i z_{ij} \leq H_j \qquad \forall j \in m \tag{4}$$

$$n_j \in n,\ z_{ij} \in \{0,1\},\ \ \forall i \in n,\ \forall j \in m \tag{5}$$

Where n is the set of customers, m is the set of facilities, n_j is the number of customers served by facility j, p_i is the demand of customer i and H_j is the capacity of facility j (in this case, all facilities have the same capacity, thus, $H_j = H$). As most of the metric distances are non-linear, (1) defines the non-linear objective function which consists on minimizing the total distance between each customer and the facility where the customer is assigned. (2) and (3) are restrictions that define that each customer is only assigned to one facility and provides the number of customers assigned to each facility respectively. (4) defines that the total demands of the customers assigned to a facility j must not exceed its capacity. Finally, (5) define the decision variable z_{ij} and the upper limits for the number of customers assigned to each facility (n_j).

In terms of the ellipsoidal Earth, as mentioned in Section 2.1, locations are expressed in latitude (λ) and longitude (ϕ) coordinates. Thus, (x_j^*,y_j^*) → (ϕ_j^*, λ_j^*) and d_{ij} → s_{ij} where s_{ij} is the arc length on the ellipsoidal Earth (geodetic distance) between the facility j located at (ϕ_j^*, λ_j^*) and the customer i located at (ϕ_i, λ_i). This leads to the following updated objective function for the capacitated multi-facility Weber problem on the ellipsoid:

$$Minimize_{\phi_j^*,\lambda_j^*} = \sum_{i=1}^{n}\sum_{j=1}^{m} z_{ij} s_{ij} \tag{6}$$

While restrictions (2) to (5) do not need further adaptation for the ellipsoidal model, the following restrictions are added to keep consistency to the search space on the ellipsoid:

$$\frac{(e\cos\phi_j^*\cos\lambda_j^*)^2}{e^2} + \frac{(f\cos\phi_j^*\sin\lambda_j^*)^2}{f^2} + \frac{(g\sin\phi_j^*)^2}{g^2} = 1 \tag{7}$$

$$-\frac{\pi}{2} < \phi_j^* \leq \frac{\pi}{2}, -\pi < \lambda_j^* \leq \pi \tag{8}$$

Where e is the major semi-axis of the ellipsoid, and f and g are the minor semi-axes of the ellipsoid. Finally, an additional restriction procedure for multiple restricted zones is required. Note that this restriction applies over the coordinates (ϕ_j^*, λ_j^*) which directly affect the decision variable z_{ij}.

If a set v of restricted or forbidden circular zones with centers located at (ϕ_f^*, λ_f^*) and radius r_f exist, then, a candidate location for a facility (ϕ_j^*, λ_j^*) is located within a restricted zone if the geodetic distance between this location at (ϕ_j^*, λ_j^*) and any center of forbidden zone (ϕ_f^*, λ_f^*) is smaller than (or equal to) to any r_f. Otherwise, the candidate solution is valid as it is out of any restricted zone. This can be expressed as:

$$s_{jf} > r_f \quad \forall j \in m,\ \forall f \in v \tag{9}$$

This restriction defines that all geodetic distances between the centers of the restricted zones and the facilities must be larger than r_f to ensure compliance of the forbidden zones.

4. Assessment of the Multi-Facility Location Model

4.1 Solving Method

As the multi-facility location problem is an NP-hard problem, the use of a meta-heuristic to provide suitable solutions was considered. The extended GRASP capacitated k-means clustering (GRASP-CKMC) algorithm presented by Caballero et al. [26] was considered for the purposes of the present work.

The GRASP-CKMC algorithm provided suitable solutions for the capacitated centered clustering problem (CCCP) which is a well-known multi-facility location problem (average error < 5.0% for large well known instances). However, the CCCP is different from the multi-facility Weber problem because, instead of locating the facilities at the locations of minimum distance to customers, the CCCP locates the facilities at average locations (centroids) between the assigned customers. Other differences of the CCCP and the GRASP-CKMC algorithm are that restrictions on the locations for facilities are not considered and the number of facilities is a decision variable.

Hence, changes were performed to adapt the GRASP-CKMC algorithm to the present work. These changes are the following:

a) the minimum number of facilities is estimated based on a lower bound defined by:

$$m_{LB} = \frac{\sum_{i=1}^{n} p_i}{H} \quad (10)$$

Note that this approximation considers that demands can be partially served by a facility. As restriction (2) defines that the demand of a customer must be served by a single facility, the complying number of facilities may be larger than m_{LB}. For this work, the number of facilities is considered as $m = m_{LB} + 10$.

b) For a set of assigned customers to facility j (performed by the GRASP-CKMC) the location of minimum distance of the facility (ϕ_j*, λ_j*) is estimated by means of a micro genetic algorithm (μGA). Figure 2 presents the structure of this algorithm.

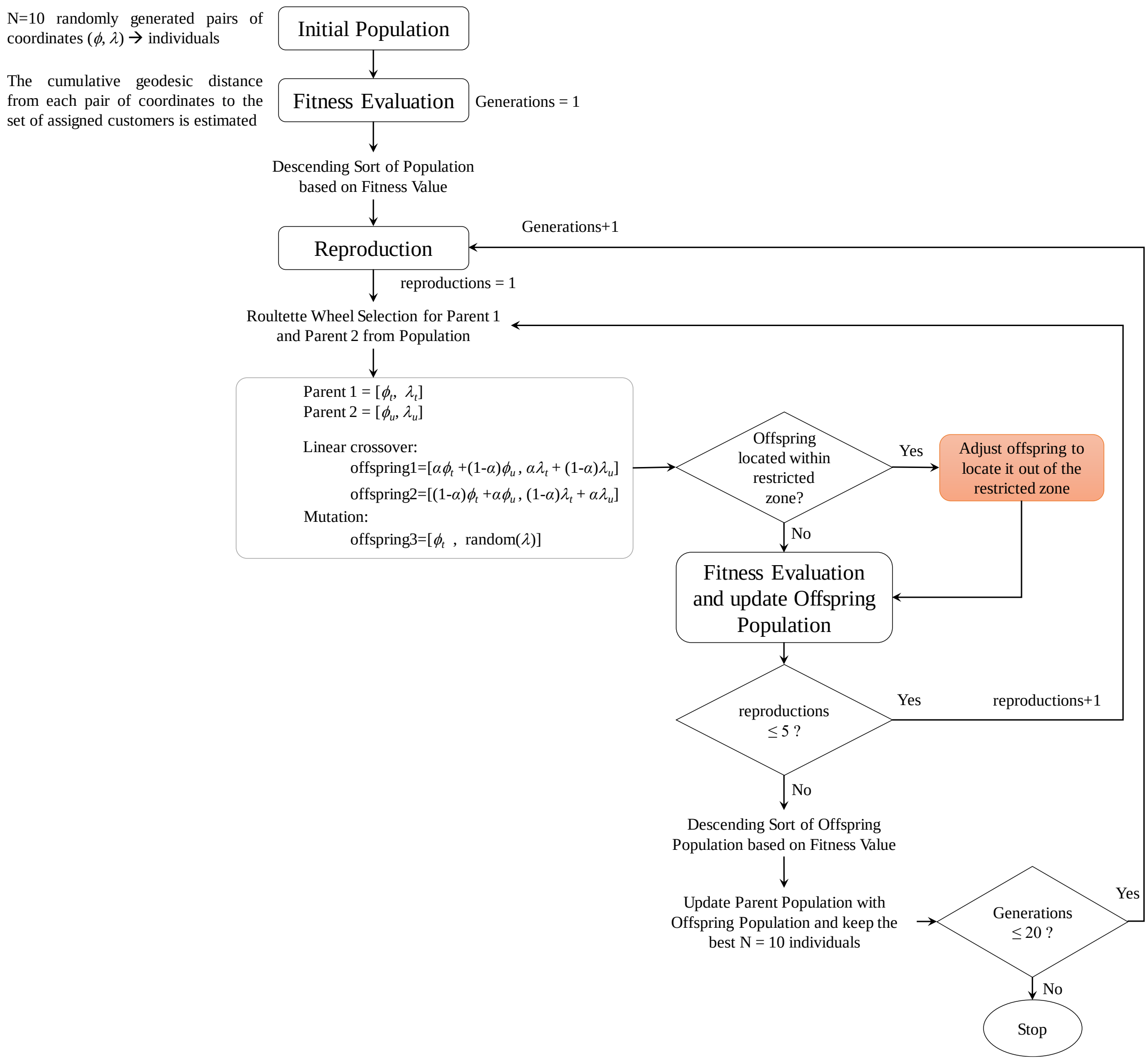

Figure 2. Structure of the μGA with restricted zones.

c) As presented in Figure 2, the searching mechanism of the μGA for (ϕ_j*, λ_j*) must comply with zone restrictions. When a candidate solution for (ϕ_j*, λ_j*) is generated through the reproduction operators of the μGA its suitability is verified to comply with restriction (9).
If compliance is not achieved, then the candidate solution is adjusted to comply with restriction (9). This is performed by "projecting" the non-compliant location within the restricted zone to a location over its perimeter (out of the restricted zone). This is performed as presented in Figure 3.
If a candidate solution (ϕ, λ) generated by the reproduction or initialization procedure of the μGA is within the perimeter of a restricted zone with center at (ϕ_f*, λ_f*) and radius r_f, the vector representing its projecting direction can be obtained as:

$$\boldsymbol{V} = [\phi - \phi_f^*, \lambda - \lambda_f^*] \tag{11}$$

The length of this vector (i.e., $|\boldsymbol{V}|$) can be obtained as the geodetic (ellipsoidal) distance between (ϕ, λ) and (ϕ_f*, λ_f*). Then, the required distance to move (ϕ, λ) to the limits of the restricted zone over the direction of $\boldsymbol{V}$ can be obtained as r_f - $|\boldsymbol{V}|+\beta$, where β is a very small distance value to ensure projection out of the restricted zone. Finally, the adjusted location of (ϕ, λ) can be obtained as:

$$(\phi, \lambda)' = (\phi, \lambda) + \frac{\boldsymbol{V}}{|\boldsymbol{V}|}\left(r_f - |\boldsymbol{V}| + \beta\right) \tag{12}$$

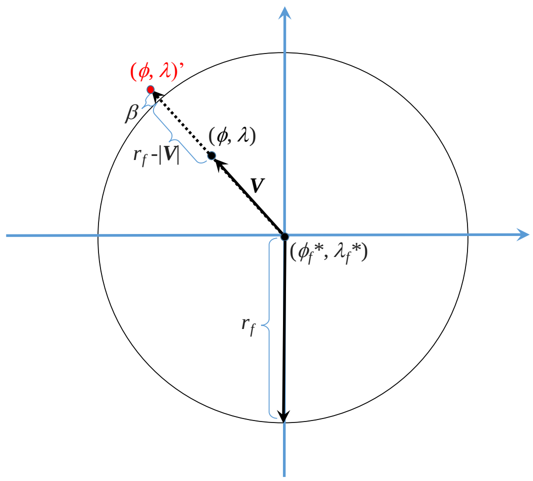

Figure 3. Adjustment of solution to locate it out of the restricted zone.

The adapted GRASP-CKMC was implemented in MATLAB R2018a. The hardware was a HP Z230 Workstation with Intel Xeon CPU E3-1240 v3 at 3.40 GHz and 8 GB RAM.

4.2 Test instance

For testing purposes with real data an instance with 500 location points was developed. Demand data for each point was randomly generated and the coordinates were considered in radians. These locations are presented in Table 1. Note that this data can be used also for benchmark purposes.

The number of restricted zones was considered as equal to the number of required facilities which was set to 60 according to $m = m_{LB} + 10$.

Table 1. Test instance with 500 geographic locations.

i	λ	ϕ	p_i	i	λ	ϕ	p_i	i	λ	ϕ	p_i	i	λ	ϕ	p_i	i	λ	ϕ	p_i
1	0.3879	-1.7079	125	101	0.9731	0.6465	56	201	0.2238	1.3929	172	301	0.7483	1.2843	34	401	0.5565	0.1043	68
2	0.7385	-1.2411	109	102	0.9750	0.6340	184	202	0.1922	1.3409	111	302	0.9665	0.1810	193	402	0.6411	0.0538	102
3	0.7385	-1.2451	148	103	0.9441	0.3212	124	203	0.1531	1.3415	67	303	0.9949	0.1732	51	403	0.6009	0.1531	88
4	0.6193	-1.7062	105	104	0.9444	0.3197	38	204	0.1434	1.3541	68	304	0.9830	0.1488	134	404	0.2201	-0.1398	45
5	0.6509	-2.1310	153	105	0.9718	0.3695	156	205	0.1893	1.3511	50	305	0.6224	-0.0963	71	405	0.2528	-0.0735	72
6	0.3527	-1.5270	104	106	0.8227	0.3013	20	206	0.2560	1.3531	74	306	0.6153	-0.1063	180	406	0.3150	0.0105	156
7	0.3329	-1.7157	92	107	0.8216	0.2683	64	207	0.3026	1.3399	76	307	0.5924	-0.1128	40	407	0.5339	0.0498	194
8	0.7951	-1.2893	163	108	0.8274	0.2772	67	208	0.3987	1.5614	165	308	0.4206	-0.2327	1	408	0.5577	0.0927	192
9	0.7627	-1.3849	74	109	0.7702	0.3446	131	209	0.4066	1.5687	9	309	0.2977	-0.2430	98	409	0.0137	0.4235	95
10	0.8437	-1.7519	25	110	0.7442	0.4046	195	210	0.3906	1.6026	20	310	0.2252	-0.2615	171	410	-0.0604	0.3917	125
11	0.1795	-1.4816	69	111	0.7356	0.4306	24	211	0.4045	1.5937	52	311	0.1811	-0.2116	112	411	-0.1619	-0.7465	157
12	0.1775	-1.1879	199	112	0.7736	0.4143	162	212	0.4346	1.6027	21	312	0.1333	-0.1536	168	412	-0.0662	-0.6735	66
13	0.5136	-1.7210	17	113	0.8203	0.5012	25	213	0.4144	1.6169	110	313	0.1072	-0.1052	198	413	-0.0907	-0.6530	21
14	0.1087	-1.3190	72	114	0.8330	0.4767	15	214	0.3999	1.6346	30	314	0.1858	-0.0836	148	414	-0.1034	-0.6158	107
15	0.7390	-1.4517	192	115	0.8818	0.5271	197	215	0.3789	1.6725	25	315	0.1098	-0.0285	42	415	-0.1445	-0.6147	120
16	0.3249	-1.6697	157	116	0.8992	0.5355	93	216	0.2951	1.6760	158	316	0.0986	-0.0039	59	416	-0.1921	-0.6500	60
17	0.4511	-1.7032	60	117	0.9358	0.4769	126	217	0.2799	1.6458	48	317	0.1494	0.0168	158	417	-0.2338	-0.8113	13
18	0.4079	-1.9160	130	118	0.9537	0.4338	184	218	0.2794	1.6496	57	318	0.1253	0.0342	148	418	-0.2744	-0.8391	37
19	0.5549	-2.0351	138	119	1.0355	0.4318	87	219	0.2894	1.6560	17	319	0.1694	0.0288	147	419	-0.2851	-0.8556	183
20	0.8132	-2.1052	164	120	1.0202	0.4163	1	220	0.2786	1.6626	35	320	0.1295	0.0657	93	420	-0.2914	-0.8630	110
21	0.6840	-2.0766	78	121	1.0224	0.4053	187	221	0.2801	1.6632	63	321	0.2275	0.0900	104	421	-0.3474	-0.7698	129
22	0.6912	-1.9345	141	122	1.0196	0.3958	77	222	0.2092	1.8098	162	322	0.2044	0.1931	12	422	-0.2209	-1.0547	38
23	0.1733	-1.4686	35	123	1.0184	0.3855	39	223	0.2437	1.8820	121	323	0.0989	0.1517	173	423	-0.3477	-1.0609	132
24	0.1549	-1.3930	143	124	1.0263	0.3950	85	224	0.1751	1.8452	182	324	0.0704	0.1688	39	424	-0.3953	-1.0496	89
25	-0.6048	-1.0223	103	125	1.0302	0.3942	88	225	0.0548	1.7736	150	325	0.1850	0.2488	104	425	-0.4279	-1.0241	29
26	0.7778	-1.1394	143	126	1.0276	0.4004	168	226	-0.0550	1.9947	81	326	0.0457	0.2681	138	426	-0.4415	-1.0065	182
27	0.9347	-1.9857	52	127	1.0299	0.4061	160	227	-0.0332	2.0278	171	327	-0.0511	0.1916	158	427	-0.5489	-1.0109	196
28	0.5960	-2.0496	55	128	1.0301	0.4039	176	228	-0.0213	2.0238	186	328	0.0200	0.2302	105	428	-0.5804	-1.0135	74
29	-0.2883	-1.1902	176	129	1.0303	0.4102	151	229	-0.0021	2.0403	8	329	-0.0730	0.2316	122	429	-0.6077	-0.9840	187
30	0.4035	-1.4363	6	130	1.0334	0.4138	137	230	-0.0043	2.0213	86	330	0.0282	0.2798	19	430	-0.5999	-0.9650	61
31	0.3890	-1.4717	139	131	1.0474	0.1923	79	231	0.0116	1.9692	64	331	-0.1131	0.2933	88	431	-0.6095	-1.0121	125
32	0.6123	-1.8643	159	132	1.1099	0.2678	13	232	0.0344	1.9712	36	332	0.0643	0.4863	62	432	-0.6090	-1.0489	93
33	-0.3999	-0.7640	84	133	1.1828	0.4177	156	233	0.0686	1.9950	155	333	-0.1598	0.4499	46	433	0.7251	-0.1474	3
34	-0.0649	-0.9380	13	134	1.1546	0.6695	180	234	0.0388	1.9469	64	334	-0.1545	0.2306	189	434	0.7184	-0.1511	45
35	0.6787	-1.3440	156	135	1.0628	0.8682	37	235	0.0194	1.9249	147	335	-0.2559	0.3086	31	435	0.7161	-0.1509	178
36	0.5468	-1.9819	138	136	1.1781	1.1162	115	236	0.0025	1.9361	163	336	-0.3556	0.2609	198	436	0.7297	-0.1184	41
37	0.2769	-1.6945	102	137	1.0100	0.9787	171	237	0.0006	1.9442	91	337	-0.4655	0.2836	78	437	0.7215	-0.1219	188
38	0.5011	-1.8526	69	138	0.5562	0.9273	122	238	0.0762	2.0510	103	338	-0.5603	0.3196	11	438	0.6948	-0.1327	195
39	0.4191	-1.8269	194	139	0.6790	1.0078	159	239	0.0898	2.0688	23	339	-0.5082	0.4552	118	439	0.6732	-0.1393	57
40	0.5550	-1.8577	155	140	0.7205	1.2067	61	240	0.0879	2.0626	112	340	-0.4496	0.4897	139	440	0.6618	-0.1475	109
41	0.1913	-1.3061	26	141	0.7539	1.3065	120	241	0.1026	2.0583	45	341	-0.4132	0.3975	182	441	0.6623	-0.1329	107
42	-0.6038	-1.0195	27	142	0.8930	1.2445	95	242	0.0231	2.0002	164	342	-0.3518	0.4501	38	442	0.6551	-0.1482	64
43	0.2699	-1.5778	111	143	0.9626	1.0669	153	243	-0.0229	2.0042	46	343	-0.3153	0.4857	101	443	0.6483	-0.1506	16
44	0.2346	-1.5544	4	144	0.8896	0.9062	165	244	-0.0443	1.9713	187	344	-0.3110	0.5396	73	444	0.6480	-0.1339	43
45	0.1585	-1.3892	53	145	0.8774	0.9958	36	245	-0.0461	1.9466	138	345	-0.4071	0.6170	5	445	0.7436	-0.0976	106
46	0.3351	-1.7486	64	146	0.9381	1.1603	69	246	-0.0430	1.9368	10	346	-0.2865	0.5728	174	446	0.7149	-0.0996	136
47	-0.0033	-1.3713	152	147	0.8792	1.3997	37	247	-0.0344	1.9350	23	347	-0.2291	0.6804	12	447	0.6889	-0.1122	125
48	-0.1592	-1.2663	23	148	0.7543	1.3380	63	248	-0.0062	1.9616	51	348	-0.2436	0.5873	12	448	0.6523	-0.1052	89
49	-0.1920	-1.0923	47	149	0.7508	1.3825	152	249	0.2381	2.1503	171	349	-0.2586	0.4323	145	449	0.6318	-0.0950	93
50	0.0490	-1.0606	74	150	0.7589	1.3990	71	250	0.2455	2.1355	200	350	-0.1787	0.5436	3	450	0.6371	-0.0861	22
51	0.7620	-1.3893	32	151	0.7493	1.4043	4	251	0.2453	2.1165	84	351	-0.1356	0.6224	71	451	0.7479	-0.0473	30
52	0.6889	-1.3113	4	152	0.7486	1.3828	190	252	-0.5575	2.0221	14	352	-0.1108	0.6365	43	452	0.7390	-0.0651	195
53	0.8170	-1.7599	86	153	0.6603	1.3076	71	253	-0.5275	2.0544	145	353	-0.0443	0.5793	144	453	0.7273	-0.0647	48
54	0.8108	-1.8345	48	154	0.3978	1.0046	158	254	-0.4712	2.0130	94	354	0.0485	0.5623	102	454	0.7270	-0.0162	166
55	0.8173	-1.9618	64	155	0.4353	0.9607	124	255	-0.4320	2.0393	120	355	0.0484	0.5629	92	455	0.7264	0.0102	168
56	0.6305	-2.0151	48	156	0.4381	0.9621	179	256	-0.3785	2.0090	185	356	0.1189	0.5177	104	456	0.7182	0.0190	97
57	0.7826	-2.1424	93	157	0.4992	0.8149	87	257	0.8359	1.8633	189	357	0.1505	0.5768	34	457	0.7168	0.0180	93
58	0.9815	-2.0488	31	158	0.4315	0.8074	185	258	0.8634	1.8481	113	358	0.1274	0.3189	142	458	0.7163	0.0127	154
59	0.9944	-1.7713	190	159	0.4271	0.6890	34	259	0.8663	1.7468	196	359	0.1409	0.3907	3	459	0.7134	0.0123	160
60	-0.5760	-1.2487	2	160	0.3741	0.6930	43	260	0.8757	1.7273	57	360	-0.0227	0.6407	62	460	0.7107	0.0122	186
61	-0.6797	-1.1901	24	161	0.3184	0.7414	185	261	0.8840	1.7291	31	361	0.0408	0.6627	161	461	0.7089	0.0101	35
62	-0.6758	-1.0878	99	162	0.2538	0.7744	88	262	0.8568	1.9872	120	362	0.0017	0.7015	77	462	0.7078	0.0092	128
63	-0.6258	-1.0912	53	163	0.2594	0.7747	62	263	0.6971	2.0266	26	363	0.0408	0.7379	60	463	0.6878	-0.0088	26
64	-0.6210	-1.1268	126	164	0.2313	0.7767	42	264	0.6640	1.9936	6	364	-0.0062	0.7423	89	464	0.6680	-0.0123	114
65	-0.5839	-1.2342	11	165	0.2360	0.8151	104	265	0.5434	1.9200	183	365	0.0673	0.7952	116	465	0.6598	-0.0148	67
66	-0.6635	-1.0078	3	166	0.2514	0.8246	121	266	0.4892	1.6099	41	366	0.1467	0.8460	22	466	0.6565	-0.0179	122
67	-0.5881	-0.9332	36	167	0.2933	0.8182	129	267	0.6071	1.4640	76	367	0.1667	0.7685	185	467	0.6629	-0.0203	11
68	-0.5493	-0.9752	158	168	0.3232	0.8864	19	268	0.7369	1.7820	175	368	0.1970	0.8914	21	468	0.7035	-0.0658	179
69	-0.4450	-0.8627	109	169	0.3407	0.9524	112	269	0.7751	1.9202	19	369	0.0942	0.6679	59	469	0.7033	-0.0671	146
70	-0.4133	-0.8181	191	170	0.3700	0.9615	147	270	0.7757	1.9387	128	370	0.1137	0.7591	195	470	0.6753	-0.0719	40
71	-0.3999	-0.7632	29	171	0.4957	0.6370	164	271	0.7513	1.9386	83	371	0.1612	0.7112	189	471	0.6805	-0.0328	14
72	-0.2251	-0.6706	169	172	0.5474	0.6396	146	272	0.6235	2.2231	72	372	0.1965	0.6428	144	472	0.6728	-0.0524	89
73	-0.2118	-0.6713	87	173	0.5335	0.6321	120	273	0.6378	2.2261	39	373	0.2356	0.6881	15	473	0.6671	-0.0597	98
74	-0.2048	-0.8573	172	174	0.6085	0.6643	174	274	0.6573	2.2094	75	374	0.2129	0.7440	57	474	0.6740	-0.0723	137
75	-0.0234	-0.8481	54	175	0.6550	0.3936	14	275	0.6042	2.2080	87	375	0.2020	0.7526	188	475	0.6698	-0.0848	170
76	0.0002	-0.8976	120	176	0.7308	0.0486	186	276	0.5840	2.2087	65	376	0.2908	0.6714	67	476	0.6594	-0.0669	110
77	0.6682	-1.6525	144	177	0.7364	0.0380	165	277	0.5728	2.2800	104	377	0.2637	0.6558	158	477	0.6963	-0.0968	47
78	0.6256	-1.5827	178	178	0.7430	-0.0076	38	278	0.5912	2.2814	144	378	0.2299	0.5262	191	478	0.6992	-0.0381	124
79	0.6205	-1.7169	37	179	0.8527	0.0397	185	279	0.6053	2.3642	90	379	0.2251	0.4096	29	479	0.7247	0.0058	97
80	0.5904	-1.5834	89	180	0.8389	-0.0524	141	280	0.5899	2.3640	151	380	0.3338	0.5304	6	480	0.7441	0.0037	158
81	0.7821	-1.6416	102	181	0.7253	0.0207	34	281	0.6225	2.4365	194	381	0.3334	0.6510	78	481	0.7524	-0.0026	129
82	0.8914	-1.9923	83	182	0.3346	1.2728	129	282	0.6317	2.4009	80	382	0.3629	0.5295	104	482	0.7042	-0.0195	3
83	0.9284	-1.9950	40	183	0.3437	1.2694	90	283	0.6625	2.4303	77	383	0.2115	0.2624	191	483	0.7010	-0.0216	44
84	0.9454	-1.8225	154	184	0.3469	1.2727	109	284	0.6937	2.4606	38	384	0.1596	0.3205	194	484	0.6999	-0.0263	10
85	0.9317	-1.1407	181	185	0.3486	1.2701	191	285	0.7089	2.4476	39	385	0.2320	0.3435	166	485	0.6933	-0.0333	180
86	0.8433	-0.9969	50	186	0.3539	1.2718	2	286	0.7443	2.4612	117	386	0.3001	0.3766	25	486	0.6617	-0.0526	142
87	0.8096	-1.1022	164	187	0.3693	1.2685	18	287	0.7467	2.4905	88	387	0.3723	0.2968	106	487	0.6590	-0.0559	172
88	0.8230	-1.1969	136	188	0.3698	1.2899	175	288	0.7636	2.4812	67	388	0.2962	0.1393	135	488	0.6537	-0.0575	117
89	0.7898	-1.3279	17	189	0.3887	1.2348	189	289	0.7655	2.5056	8	389	0.3668	0.2147	40	489	0.6576	-0.0623	42
90	0.7301	-1.5361	67	190	0.4205	1.2607	57	290	0.7561	2.5403	48	390	0.2938	0.2317	175	490	0.6496	-0.0711	73
91	0.7660	-1.6797	188	191	0.4605	1.2877	28	291	0.6185	1.6442	27	391	0.5246	0.5443	174	491	0.6762	-0.1611	142
92	0.6803	-1.8814	84	192	0.3794	1.3823	55	292	0.7089	1.4477	188	392	0.4453	0.5089	94	492	0.6865	-0.1484	49
93	0.7375	-1.5570	23	193	0.3817	1.3823	181	293	-0.1012	2.3423	72	393	0.4174	0.6189	151	493	0.7045	-0.1406	36
94	0.6437	-1.8234	115	194	0.3039	1.3660	197	294	-0.1139	2.3450	84	394	0.5469	0.4745	53	494	0.7071	-0.1395	181
95	0.5970	-1.8663	3	195	0.2917	1.4149	199	295	-0.1365	2.4122	94	395	0.4719	0.2514	174	495	0.7185	-0.1361	29
96	0.5953	-1.8719	51	196	0.2956	1.4345	80	296	-0.1390	2.4240	142	396	0.5443	0.2886	154	496	0.6758	-0.1397	3
97	0.5426	-1.8853	38	197	0.2976	1.4356	55	297	-0.0925	2.4679	196	397	0.4481	0.3674	83	497	0.6648	-0.1296	183
98	0.5564	-1.7875	158	198	0.2981	1.4350	35	298	-0.1096	2.5455	185	398	0.4221	0.4030	184	498	0.7090	-0.1445	30
99	0.4835	-1.7041	147	199	0.3013	1.4358	87	299	-0.1170	2.5656	19	399	0.4447	-0.0762	113	499	0.7200	-0.1390	67
100	0.4339	-1.7382	89	200	0.2512	1.3876	62	300	0.9257	1.0745	175	400	0.4342	0.1473	18	500	0.7083	-0.1476	36

4.3 Results

As previously mentioned, 60 facilities were considered to serve the demands of the 500 customer locations presented in Table 1. First, the multi-facility Weber problem on the ellipsoid without zone restriction was solved with the adapted GRASP-CKMC. The general assignment which is presented in Figure 4 led to a total coverage distance of 4,259.07364 km.

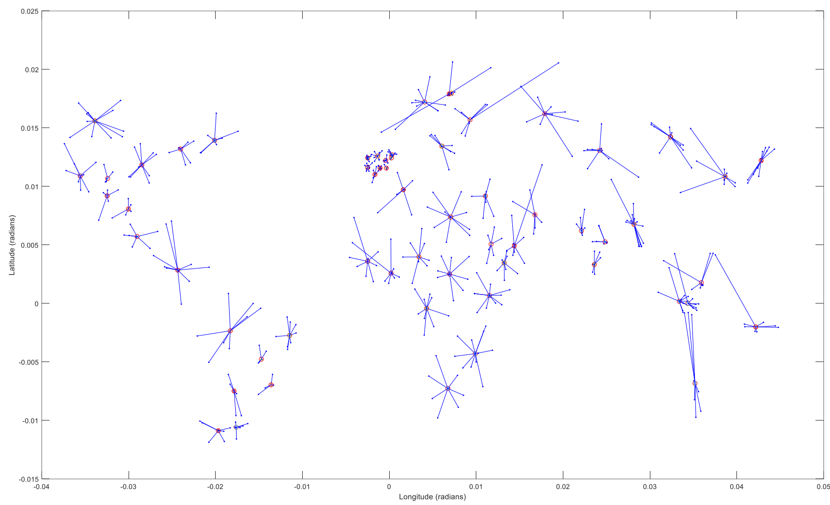

Figure 4. Solution of the multi-facility Weber problem on the ellipsoidal Earth without zone restriction

Second, with the most suitable locations for the facilities the multi-zone restriction was established. For this case, the locations of the facilities were determined as forbidden (restricted) and random radius were assigned to each facility to define the size of the restricted zone. When solving the instance with the multi-zone restriction the adapted GRASP-CKMC led to the general assignment presented in Figure 5 with a total coverage distance of 4,342.91858 km. This represents an increase of just (4,342.91858/4,259.07364 – 1) × 100 = 1.968 % when compared to the unrestricted problem. Thus, even if the most suitable locations are no longer allowable, the adapted GRASP-CKMC algorithm can provide very efficient solutions. Figure 6 presents a more detailed close-up of the difference between solutions for the unrestricted and the restricted problems.

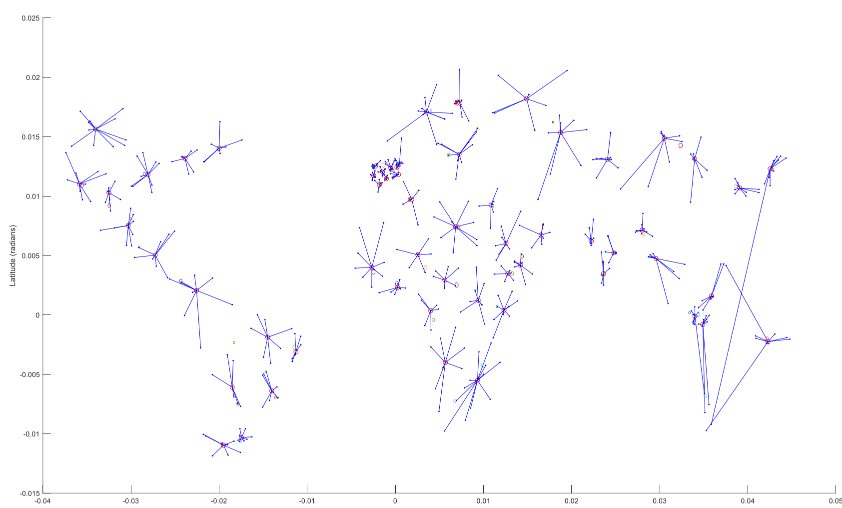

Figure 5. Solution of the multi-facility Weber problem on the ellipsoidal Earth with zone restriction

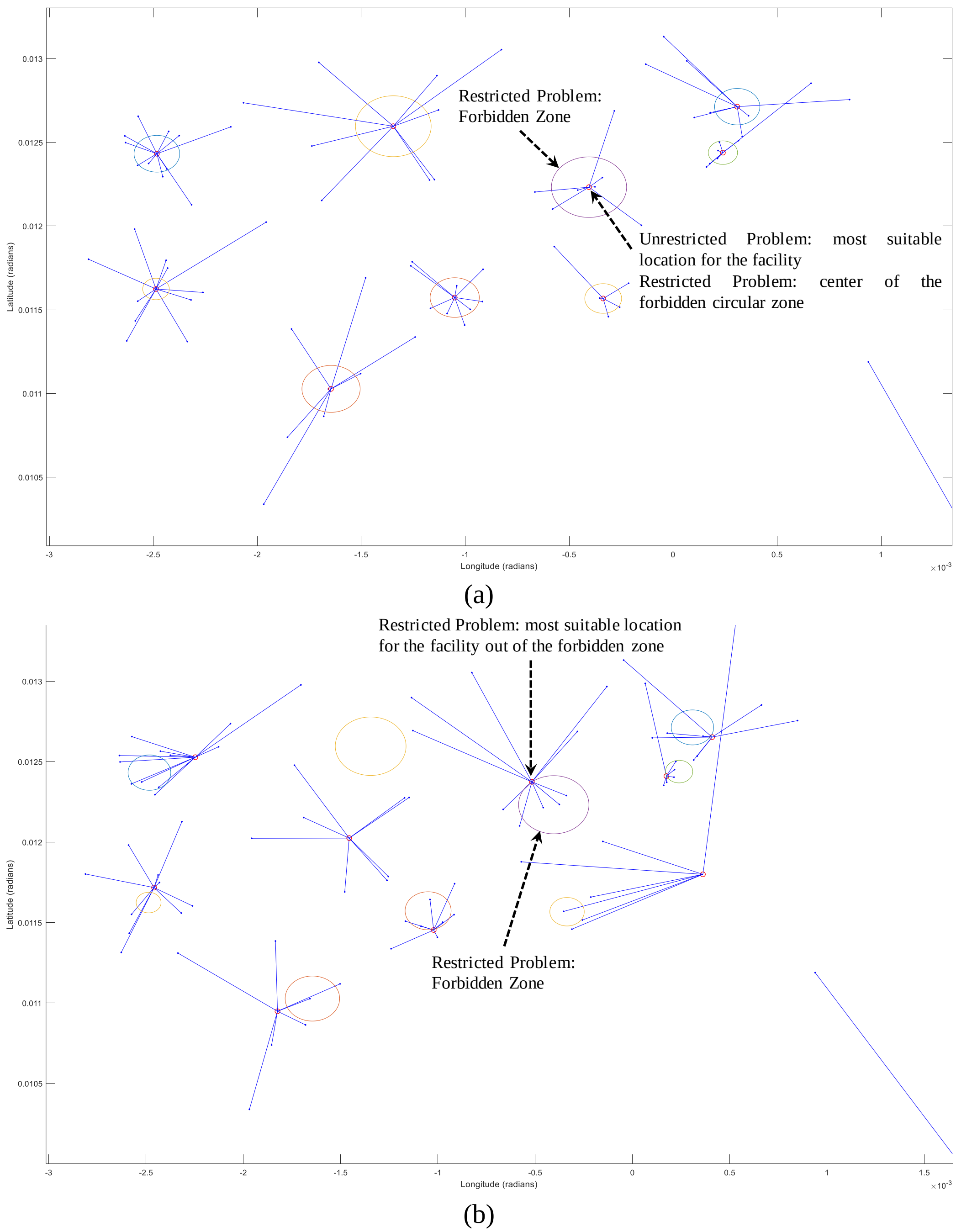

Figure 6. Comparison of solutions for the (a) unrestricted multi-facility Weber problem and the (b) restricted multi-facility Weber problem on the ellipsoidal Earth.

In Figure 6 (a) the unrestricted problem is solved and the facilities are located in the minimum distance location. Figure 6 (b) presents the solutions for the restricted problem if the previous locations for the facilities are restricted or forbidden. As presented, all facilities are located out of the restricted zones and re-assignment of customers is appropriately performed.

5. Conclusions

The location of facilities can determine the economic success of businesses and industries. Thus, uncertainty in the decision process to determine the most appropriate location can lead to negative economic performance.

In practice, uncertainty is increased when the most suitable location cannot be selected. In such case, more appropriate methods are required to identify the best alternative location in presence of this restriction.

This problem has been addressed in the specialized literature, however, these consider only a single location restriction with standard (non-geographical) data on the plain model of the Earth surface which leads to Euclidean distances.

The present work extends on this context by addressing the multi-facility Weber problem considering the ellipsoidal model of the Earth surface which leads to a more accurate representation of geographical distances. In addition, circular multi-zone restrictions were considered.

Besides extending on the mathematical modelling of the multi-facility Weber problem to include multi-zone restrictions, a method was proposed to adjust candidate solutions for this problem to locate them out of the restricted zones. This method can be integrated into other solving methods such as meta-heuristics to integrate this restriction into other facility location problems.

The results obtained with a large instance support the functionality of the model and the adjustment method, providing solutions with an error of 1.96% when compared to the unrestricted problem. Thus, the proposed model can be used in practical cases where multiple restrictions exist on the eligible locations.

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