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M-ANFIS model to determine the urban travel time with uncertain edges

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Abstract. This paper proposes a method to determine urban travel time using imprecise variables. The model calculates values for each edge of the network utilize base a Fuzzy Logic scheme and an adaptive architecture with neural networks. For the uncertainty, the effect of three variables for each street considers; the State of the streets, Traffic zones, and Rainfall (intensity of rain) with an adaptable neural network's architecture. The membership functions experimentally are calculated, getting times closer to the real. The model evaluates the uncertainty for each of the network's edges (streets), intending to adjust the set route travel time; finally, the model updates and modifies the membership functions, making it adaptable to new scenarios. The validation is compared to the M-ANFIS travel time model with the Fuzzy Logic model, probabilistic model, and desired travel time. The mean absolute percentage error of the three models and the travel time compared, obtaining the ANFIS model a lower result.

Keywords: Neuro-fuzzy, Fuzzy Logic, Artificial Neural Network, Travel Time, Time Adjust.

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1 Introduction

The costs of a product generally vary depending on the type of product to be marketed, the manufacturing process, and the distribution process conditions generate 30% of the production cost. Today, companies invest in machines, processes, and methods that ensure a quality product delivery to the customer in time and form; this generates a need in companies to look for alternatives that improve these products' delivery. The first objective is to reach the right place in the given time [1], in the right conditions, and at a competitive cost [2]. The best route optimization solutions help delivery companies minimize driving time and reduce fuel consumption. Artificial Intelligence is a tool that helps us solve complex model problems. Heuristic methods such as Fuzzy Logic or Neural Networks are alternatives with more applications in the shortest path area [3-7]. However, in real applications, the models present uncertainty in practice, and the behavior of the algorithm is considered related to a random variable [8] where the weights of each edge are interpreted randomly [9]. Some authors integrate probabilistic elements into mathematical models, increasing their complexity to solve it [10-12]. Some works present neural networks to solve transport problems [13] and robot route planning [14,15], among two techniques' union results in methods that adapt their parameters, called Neuro-fuzzy [16-19]. This work's main objective is to apply a Neuro-fuzzy network where the membership functions are adapted and adjusted to the new scenarios, resulting in optimizing the distribution route, generating new route alternatives that ensure timely deliveries, and delivering the product at the established time. Also, there are benefits such as reduced fuel costs, reduced vehicle maintenance caused by wear, and the travel of poor routes, in the same way as a company to manage policy and ensure timely delivery. The results purchase with a Fuzzy model and a probabilistic model.

1.1 Stochastic process

A stochastic process is a collection of random variables $\{X_t\colon t \in T\}$ parameterized by a set T, called a parametric space, and values in a set S called a state space [20].

Formally, a stochastic process describes as a function of two variables $X\colon T \times \Omega \rightarrow S$ such that the pair (t, ω) is associated with the state $X(t, \omega)$, which can also write as $X_t(\omega)$. For each value of t in T, the mapping $\omega \rightarrow X_t(\omega)$ is a random variable, while for each ω in fixed Ω, the function $t \rightarrow X_t(\omega)$ is called a path or realization of the process.

1.1.1 Conditional probability

Let H be an event with positive probability for an arbitrary event A, and P(AH) the event where A and H occur simultaneously. The equation is written in (1) [21].

$$P(A|H) = \frac{P(AH)}{p(H)} \quad \textbf{(1)}$$

Often equation (1) is used in form (2)

$$P(AH) = P(A|H) \cdot P(H) \quad \textbf{(2)}$$

In [22], authors define the relative frequency of A conditional on B's occurrence, and from this definition, deduces the concept of conditional probability. Indicating "where $A \cap B$ represents the joint occurrence event of A and B, and we assume $P(B) > 0$". Implicit in this definition is the restriction of the sample space. Doing the following properties:

- It is essential to differentiate between $P(A \cap B)$ and $P(A \mid B)$.
- A joint probability $P(A \cap B)$ is always less than the simple probabilities $P(A)$ and $P(B)$.
- A conditional probability $P(A \mid B)$ can be greater than, less than, or equal to $P(A)$.
- The sample space in the conditional probability $P(A \mid B)$ restrict to B.

1.2 Fuzzy Logic

The Fuzzy set concept is an extension of Zadeh's classical theory (1965) [23], often considered in engineering. Some concepts are neither entirely real nor completely false or are partly true and partly false. Zadeh (1973), "As complexity increases, precise statements lose their meaning, and useful statements lose precision." In mathematics and statistics, a Fuzzy variable (such as "temperature," "performance," or "maintenance") is a value that would be within a range defined by the quantitative limits and that is usefully described verbally with imprecise categories (e.g., as "high," "medium" or "low").

1.3 Neuro-fuzzy

The Adaptive Neural Fuzzy Inference System (ANFIS) is a combination of two soft computing methods: Artificial Neural Networks (ANN) and Fuzzy Logic (FL) [23]. Fuzzy Logic can change the qualitative aspects of human knowledge and ideas in the process of quantitative analysis. On the other hand, ANNs have a greater capacity in the learning process to adapt to their environment.

ANNs use to adjust membership functions from data or to enrich it by learning from examples. The ANNs reduce the error rate when determining rules in Fuzzy Logic. The term "Neuro-fuzzy Systems" applies to systems with the following properties:

- Train with a learning algorithm that modifies information from the Fuzzy System itself. The learning process does not base on knowledge but on a given data set.
- The Neuro-fuzzy System takes the form of a multilayer neural network, where the units in their nodes use T-norm or T-conorm operations, substituting the activation functions commonly used by neural networks. The first layer represents the input variables. The second contains the membership functions and Fuzzy rules that describe the expert's expertise through hidden layers and connections. The last layer represents the output variables.
- Approximate an n-dimensional function, partially defined from training data. Fuzzy rules encoded within the system represent ambiguous samples and interpret them as training data patterns.

2 Experimental procedures

2.1 Probabilistic model

2.1.1 Traffic patterns

To analyze traffic patterns, first, the area or distribution area is delimited later to compare the area's typical patterns during the week. The schedules group into three periods, morning, afternoon, and night (from 7 to 22 hours) that has a more significant influx of vehicles in the area, in five scenarios, Monday, Tuesday-Wednesday-Thursday, Friday, Saturday, considering Tuesday, Wednesday, and Thursday as scenarios with similar behaviors. The data obtain with the help of the Google Maps traffic tool; the function is approximated with an R^2 of more than 90% for the period's morning, afternoon, night (Figure 1-3). These scenarios make up the realizations of the probabilistic model, where the function associates a signal for each possible result of a random experiment

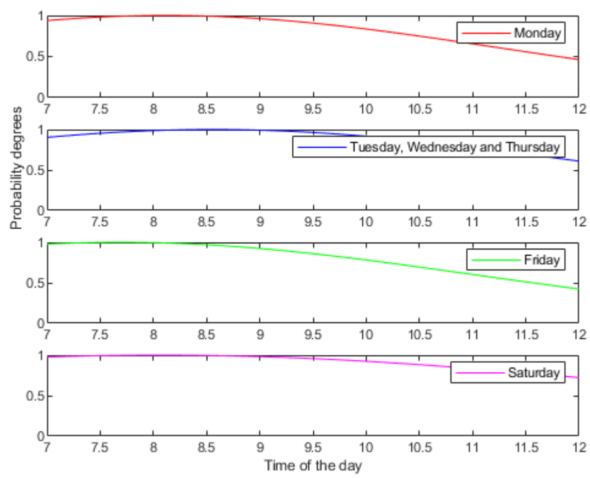


Fig. 1. Realization for the morning period.

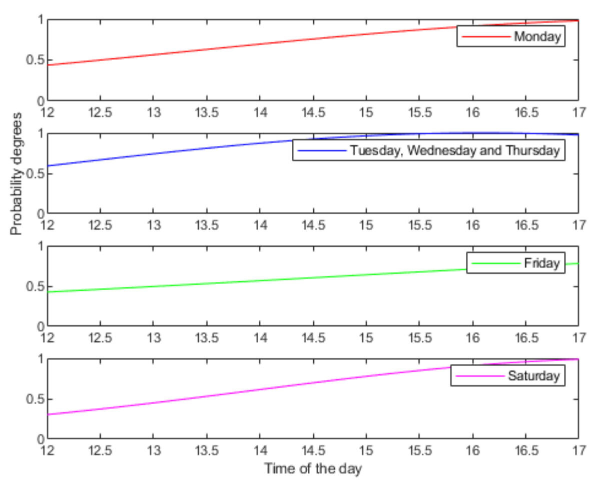


Fig. 2. Realization for the afternoon period.

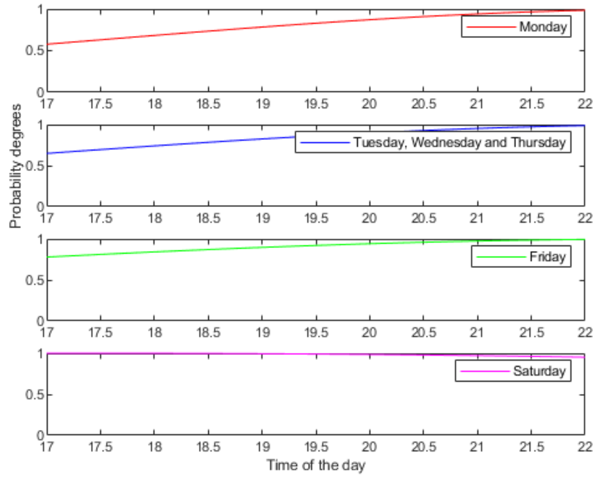


Fig. 3. Realization for the night period.

where the function associates a signal for each possible result of a random experiment where the x-axis represents the working hours of the day and the y-axis represents the traffic event in percentage: slow for values from 0% to 25%, relatively slow in the ranges from 26% to 50%, relatively fast for values between 51% to 75%, and fast in the range 75% to 100%.

2.1.2 *Rainfall*

In https://es.weatherspark.com, the data for the city of Tuxtla Gutiérrez, Chiapas obtains. The realizations for this variable determine for the 12 months of the year, excluding night hours. Figure 4 shows the realizations of some months of the year

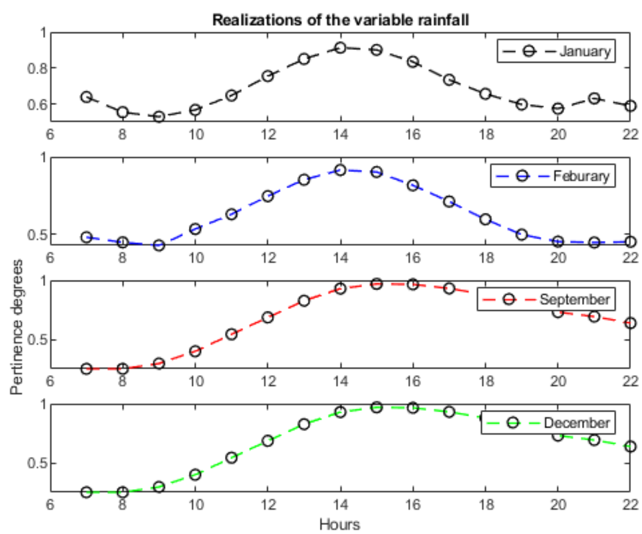


Fig. 4. Realization for the rainfall period.

where the x-axis represents the working hours of the day, and the y-axis represents the pertinence degrees for the time of day that is most likely to occur, excluding hourly accumulation of fewer than 0.25 millimeters.

However, the probability that the rain occurs in a given hour is an event independent of the amount of rain that falls at that moment in time, then two events *A* and *B* are defined, which are independent variables where the occurrence of one does not influence on the other. Mathematically they are defined in Equation 3 [24].

$$P(A \cap B) = P(A) \cdot P(B) \tag{3}$$

where; A is the event of the time of occurrence of rain and B the event of the amount of rain.

For the probability of the amount of rain at the instant of time, the data was provided by the Coordinación General del Servicio Meteorológico Nacional (CGSMN) of the Comisión Nacional del Agua (CONAGUA), registered in the observatory and the automatic meteorological station ESIME (Estación Sinóptica Meteorológica), the data are collected from 1970 to 2015 in the city of Tuxtla Gutiérrez, Chiapas, Mexico. They are classifying the amounts of rain according to CONAGUA in intervals showing: no rain for values of 0 to 0.25 mm of water, rain with values between 0.26 mm to 25 mm, heavy rain in the interval of 26 mm to 75 mm, and torrential rain for values higher than 75 mm, in Table 1, a summary of the classification and probability of the amount of rainfall present.

Table 1. Probability of amount of rain per month

	No rain 0≤ x 0<.25mm	Rain 0.25mm≤ x <25mm	Heavy rain 25mm≤ x <75mm	Torrential rain x ≥ 75mm
January	99.22%	0.70%	0.08%	0.00%
February	98.21%	1.79%	0.00%	0.00%
March	98.52%	1.40%	0.08%	0.00%
April	95.44%	4.16%	0.41%	0.00%
May	73.82%	21.60%	4.27%	0.31%
June	47.22%	43.71%	8.82%	0.25%
July	50.09%	42.63%	7.19%	0.09%
August	46.82%	44.87%	7.98%	0.33%
September	43.53%	47.07%	8.90%	0.49%
October	78.12%	19.65%	2.07%	0.16%
November	93.93%	5.75%	0.32%	0.00%
December	97.88%	2.12%	0.00%	0.00%

This variable is considered as the intensity of rain, having a duration of one hour, data that report in the literature determine the intensity for the city of Tuxtla Gutierrez, Chiapas of rain is of 65 mm/hr [25]; The procedure uses the information of isohyets of intensities, available in the Mexican Republic, making use of the pluviometry records to estimate the necessary predictions of maximum precipitation in 24 hours. Therefore, it can be applied in any site or locality in the Mexican Republic with a rainfall station, which allows obtaining the values above with absolute reliability.

2.1.3 Probabilistic adjustments determination

Before obtaining the results, it is necessary to calculate the value to which the stochastic model adjusts the travel times using the speed equation, Equation 4.

$$t = \frac{d}{v} \tag{4}$$

The distance is the length of the street from a node to the neighboring node; the speed is determined by the type of road, according to Tuxtla Gutiérrez's municipality's current regulations https://tuxtla.gob.mx/1Normatividad-vigente. This part defines main avenues and road distributors of 40 km/h maximum; on boulevards and peripherals of 50 km/h maximum; and in the bypasses, it is 70 km/h maximum; If there are no signs in urban areas, the speed is 20 km/h on streets.

The events that cause traffic are the amount of rain and rain time since the rain event is independent of the time event. In general, for any number of independent events, the probability that all the events happen is the product of the probabilities that the individual events occur (Equation 5); also, the traffic event is dependent on both, so the situation is modeled with a Bayesian network, see Equation 6.

$$P(C \cap H) = P(C) \cdot P(H) = P(D) \tag{5}$$

$$P(T_i|D) = \frac{P(D|T_i) \cdot P(T_i)}{\sum_{k=1}^{n} P(D|T_k) \cdot P(T_k)} \tag{6}$$

This equation allows us to calculate the conditional probability $P(T_i \mid D)$ of any of the event's $P(A_i)$, given D, where the variable names describe as; T=Traffic, C=Amount of rain, H=hour of rain and D= the intersection of two independent events (C∩H).

2.2 Fuzzy model

The Fuzzy System and the membership functions calculated experimentally were presented at the Congreso Mexicano de Inteligencia Artificial (COMIA) 2019 and published as a special issue of the Computing Science journal ISSN: 1870-4069, indexed in Latinindex and DBLP [26].

2.3 Neuro-fuzzy model

2.3.1 Input variables

A study to fast-food vehicle operators is performed; the study detects 52 elements. In the said analysis, those whose behavior is uncertain, more frequently, and with an important relationship in the distribution process, are determined. The proposed model consists of three variables, which modify each edge of the network's travel time, described below.

- *State of the street (S)* defines by the amount and size of obstacles that vary the vehicle's speed; it is considered diffuse interpreting a level for each state of the street.
- *Traffic* Zone (Z) certain zones are considered to generate more traffic when traveling due to school zones and shopping centers' proximity.
- *Rainfall (R)*, the intensity of rain is the amount of water that falls in the area where you drive, which causes a decrease in visibility and vehicle speed; this variable measures in ml/m^2.

2.3.2 Multilayer architecture selection

ANFIS models integrate information included within a data set with experts' knowledge, expressed in linguistic form.
An architecture based on the Mamdani-type inference mechanism (M-ANFIS) [27], Equation 7, is used.

$$\text{Rule } k = \textbf{if } R \textbf{ is } M_{1,j},\ S \textbf{ is } M_{2,j} \textbf{ and } Z \textbf{ is } M_{3,j} \textbf{ then } D_k = M_{ok}\ (\mu; FA_{ok}) \tag{7}$$

The Mamdani [28] FIS architecture is illustrated in Figure 5. The rule base in this work is written in Equation 8.

$$
\begin{aligned}
&1.\quad \textbf{if } (R \textbf{ is } M_{1,1}), (S \textbf{ is } M_{2,1}) \textbf{ and } (Z \textbf{ es } M_{3,1}) \textbf{ then } D_1 = M_{o1}\ (\mu;\ FAo_1)\\
&2.\quad \textbf{if } (R \textbf{ is } M_{1,1}), (S \textbf{ is } M_{2,1}) \textbf{ and } (Z \textbf{ es } M_{3,2}) \textbf{ then } D_2 = M_{o2}\ (\mu;\ FA_{o2})\\
&\qquad\vdots\\
&53.\quad \textbf{if } (R \textbf{ is } M_{1,6}), (S \textbf{ is } M_{2,3}) \textbf{ and } (Z \textbf{ es } M_{3,2}) \textbf{ then } D_{53} = M_{o53}\ (\mu;\ FA_{o53})\\
&54.\quad \textbf{if } (R \textbf{ is } M_{1,6}), (S \textbf{ is } M_{2,3}) \textbf{ and } (Z \textbf{ es } M_{3,3}) \textbf{ then } D_{54} = M_{o54}\ (\mu;\ FA_{o54})
\end{aligned}
\tag{8}
$$

where, R, S, Z corresponds to the input variables, Rainfall, State of the street, and Traffic zone. M_{ij} represents the *j-th* membership function of the *i-th* input, D_k and M_{ok} the output of the *k-th* rule, and the *k-th* FM of output. FA_{ok} are the consequent parameters that characterize the forms of the output membership functions, called Fuzzy Adjust.

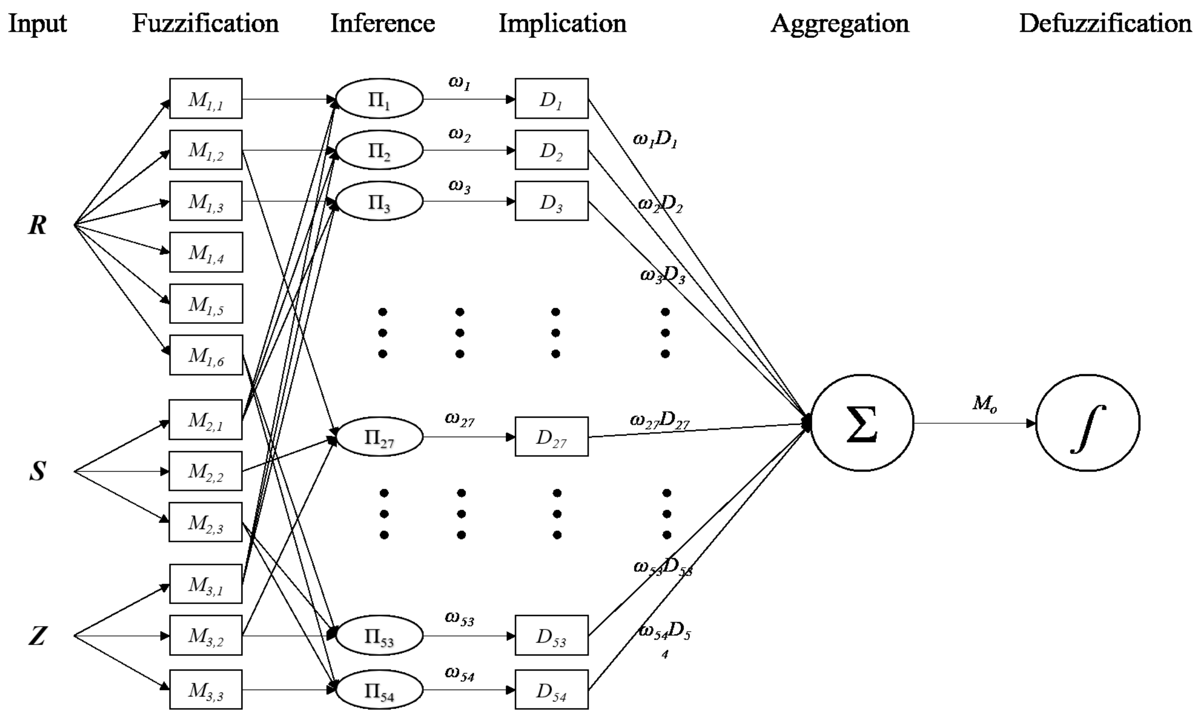


Fig. 5. Mamdani's Fuzzy inference system architecture proposal.

2.3.3 Mamdani-type architecture development

The ANFIS Mamdani model consists of 5 layers; each layer output describes below:

Layer1 (Fuzzification): The inputs of this layer, R, S, Z, represent the system's real inputs. The output means the degree of membership for which the input variable satisfies the linguistic term. This work uses membership functions calculated experimentally, described in Equations 9-13 [29].

They do not perform any processing on the variables' input values; they distribute these values to the next layer.

I. The inputs $M_{i,j}$ use Gaussian functions for *(i = 1),(j = 2; 3; 4; 5); (i = 2),(j = 2); (i = 3),(j = 2)*

$$\mu_{\{M_{i,j}\}}(x_i) = \begin{cases} e^{-k_j\ (x_i\ -m_j\ )^2}, & 0 \le x_i \ge b_j \\ 0, & b_j > x_i \end{cases} \quad \textbf{(9)}$$

II. Pseudo-exponential for input $M_{i,j}$ with *(i=2), (j=3); (i=3),(j=1,3)*

$$\mu_{\{M_{i,j}\}}(x_i) = \begin{cases} \frac{1}{1 + k_j(x_i - n_j)^2}, & a \le x_i \ge b_j \\ 0, & x_i < a_j \end{cases} \quad \textbf{(10)}$$

III. Singleton membership function for input, $M_{1,1}$

$$\mu_{\{M_{1,1}\}}(x_i{:}\, a_j) = \begin{cases} 0, & x_i = a_j \\ 1, & x_i \ne a_j \end{cases} \quad \textbf{(11)}$$

IV. Gamma for membership function, $M_{1,6}$

$$\mu_{\{M_{1,6}\}}(x_i) = \begin{cases} 1 - e^{-k_j(x_i - p_j)^2} & d_j \le x_i \ge e_j \\ 0, & 0 < d_j \end{cases} \quad \textbf{(12)}$$

V. The Sigmoidal function corresponds only to the input $M_{2,3}$

$$\mu_{\{M_{2,3}\}}(x_i) = \begin{cases} \frac{1}{1 + e^{-k_j(x_i - q_j)}}, & a \le x_i \ge b_j \\ 0, & x_i < a_j \end{cases} \quad \textbf{(13)}$$

where: a_j, b_j, d_j, e_j, k_j, m_j, n_j, p_j, q_j, are the parameters that characterize the membership functions of each input variable.

Layer 2 (Inference Layer or Rules Layer): In this layer, each node corresponds to a linguistic label (good, regular, bad, among others), for each of the input variables of each rule ω_k is calculated, considered as the weighting factor or trigger force, it is determined by evaluating the expressions of membership of the antecedent rule.

The input values or real values transform into Fuzzy values using each variable's functions from layer 1, Equation 14.

$$\begin{aligned}\omega_1 &= \mu_{M_{1,1}}(R)\mu_{M_{2,1}}(S)\,\mu_{M_{3,1}}(Z)\\ \omega_2 &= \mu_{M_{1,2}}(R)\mu_{M_{2,1}}(S)\,\mu_{M_{3,1}}(Z)\\ \omega_3 &= \mu_{M_{1,3}}(R)\mu_{M_{2,1}}(S)\,\mu_{M_{3,1}}(Z)\\ &\vdots\\ \omega_{53} &= \mu_{M_{1,6}}(R)\mu_{M_{2,3}}(S)\,\mu_{M_{3,2}}(Z)\\ \omega_{54} &= \mu_{M_{1,6}}(R)\mu_{M_{2,3}}(S)\,\mu_{M_{3,3}}(Z)\end{aligned} \tag{14}$$

Layer 3 (Implication or Antecedent):

A consequent is a Fuzzy set represented by a membership function D_k. Two of the most used are the "AND" (minimum) method, which truncates the Fuzzy output set, and the PROD (product), which scales to the Fuzzy output set.

The degree of truth of each rule's premise is computed and applied to each rule's consequent. This result is a Fuzzy subset that assigns to each corresponding output variable. The Mamdani method or MIN-MAX inference method is applied. The MIN made between the trigger level and the membership function of the output variable indicated in the consequent.

In this layer, the implication of each FM output calculates with Equation 15.

$$M_{imp,k} = \omega_k D_k, \qquad k = 1,2,3 \ldots, 53,54 \tag{15}$$

where, $M_{imp,k}$ means the implication of the output membership function.

Layer 4 (Aggregation or Consequent):

In this layer, aggregation performs to produce a general output, $M_o(u)$, using the union or maximum operator, Equation 16.

$$M_o(u) = \sum_{k=1}^{54} M_{imp,k} \tag{16}$$

Replace Equation 15 into 16, we get 17.

$$M_o(u) = \sum_{k=1}^{54} \omega_k D_k \tag{17}$$

where, D_k represents the output "**then**," describes a membership function, replacing the pseudo-exponential membership function in Equation 17 gives 18.

$$M_o(u) = \sum_{k=1}^{54} \omega_k e^{-k(x-m)^2} \tag{18}$$

Layer 5 (Combination or Defuzzification):

The output calculates using the centroid or Center Of Gravity (COG) method that returns the area's center under the curve, described in Equation 19.

$$\mathrm{o} = \frac{\int M_o(u)\, u du}{\int M_o(u)\, du} \tag{19}$$

2.3.4 Weight update

The parameters determination of the Fuzzy inference system is of vital importance. An optimization algorithm uses to determine these parameters and minimize the error measure between the destination and the real output.

During the FIS learning process, the premise and consequent parameters tune until the desired FIS response reach.

ANFIS uses hybrid learning systems; said methods are the descending gradient and the least-squares estimator.

More specifically, the rule performs the minimization of the squared error, between the input u and the output d to adjust the weight vector $\omega = [\theta, \omega_1, \omega_2, \dots \omega_n]^T$ of the network. In summary, the objective is to obtain an optimal weight ω^* so that the squared error $\{E(\omega^*)\}$ of the entire set of samples is as low as possible. In [30], the mathematical notation is affirmed, considering an optimal weight configuration, Equation 20.

$$\mathrm{E}(\omega^*) \leq E(\omega), \qquad \forall \omega \in \Re^{n+1} \tag{20}$$

The squared error function related to the p training samples is defined in Equation 21.

$$E(\omega) = \frac{1}{2} \sum_{k=1}^{p} (d^k - u)^2 \tag{21}$$

Substituting the result of Equation 21 required to obtain the output of an ADALINE neuron, Equations 22 and 23.

$$E(\omega) = \frac{1}{2} \sum_{k=1}^{p} \left(d^k - \left(\sum_{i=1}^{n} \omega_i x_i^k - \theta \right) \right)^2 \tag{22}$$

$$E(\omega) = \frac{1}{2} \sum_{k=1}^{p} \left(d^k - \left(\omega^k x_i^k - \theta \right) \right)^2 \tag{23}$$

Thus, the expression in Equation 23 calculates the Mean Squared Error for the p training samples provided for the ADALINE training process. The next step consists of applying the gradient operator on the Mean Square Error concerning the vector ω, finding an optimal value for the quadratic error function, Equation 24.

$$\nabla E(\omega) = \frac{\partial E(\omega)}{\partial \omega} \tag{24}$$

Replace 23 into 24 you get 25.

$$\nabla E(\omega) = \sum_{k=1}^{p} \left(d^k - (\omega^T x^k - \theta)\right)(-x^k) \tag{25}$$

The ADALINE output (y) is the same sequence defined for the perceptron; the calculation expresses in 26.

$$u = \sum_{i=1}^{n} \omega_i x_i - \theta \Leftrightarrow u = \sum_{i=0}^{n} \omega_i x_i \tag{26}$$

The expression 27 is obtained by substituting 25 in 26.

$$\nabla E(\omega) = -\sum_{k=1}^{p} (d^k - u)\,(x^k) \tag{27}$$

In this step, the optimization objective is to minimize the quadratic error. The variation *Δω* to update the ADALINE weight vector is given by Equation 28.

$$\Delta\omega = -\eta \nabla E(\omega) \tag{28}$$

Substituting 27 for 28, we get the expression 29.

$$\Delta\omega = \eta \sum_{k=1}^{p} (d^k - u)x^k \tag{29}$$

Expressing the increment of *Δω* and solving for the current value, we obtain 30.

$$\omega^{current} = \omega^{previous} + \eta \sum_{k=1}^{p} (d^k - u)x^k \tag{30}$$

Simplifying the expression, ω update after presenting each *k-th* training sample, Equation 31.

$$\omega^{current} = \omega^{previous} + \eta(d^k - u)x^k, \qquad k = 1 \dots p \tag{31}$$

where:
$\omega=[\theta, \omega_1, \omega_2, \dots \omega_n]^T$, is the vector that contains the threshold and the weights. $x^k=[-1, x_1^k, x_2^k, \dots x_n^k]^T$, is the *k-th* training sample. d^k, is the *k-th* training sample. u, is the output value of the linear element. η, is a constant that defines the learning rate.

We proceed to indicate the steps to follow for the implementation of this rule.

- The parameters of each membership function are initialized with small random values.
- The training pair is selected and applied as an input vector to the network.

- The network output is calculated.
- The error between the network output and the desired output is calculated, and the value accumulated in the total error.
- Steps 1 through 4 are repeated for each vector in the training set until the set's error is low enough.

3 Case study

The desired travel time values used in this article were obtained by delimiting an area of Tuxtla Gutiérrez, Chiapas, Mexico, see Figure 6. The data were timed and recorded to calculate later the shortest route with Floyd's algorithm. For the case study, the three models analyzed using the established route's travel times.

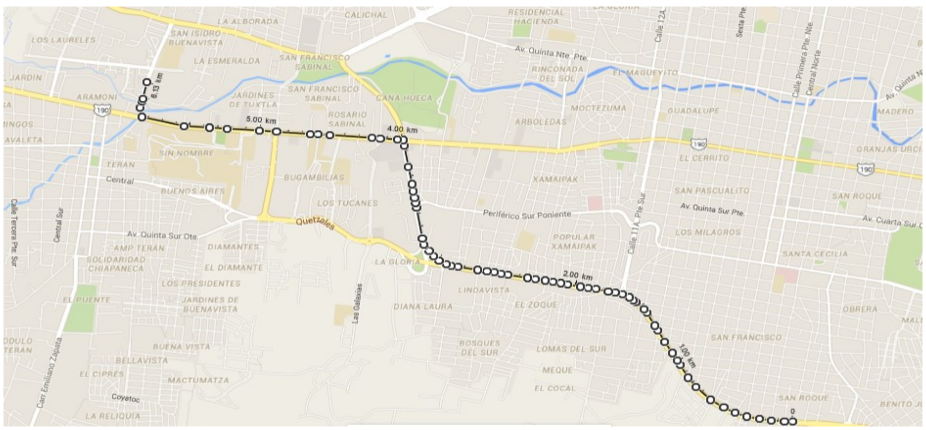


Fig. 6. Route of Tuxtla Gutiérrez, Chiapas, Mexico [31].

4 Results

The probabilistic model represents a tree diagram associating the probability corresponding to each of the branches; see Figure 7.

The model can ask questions like, "What is the probability that it finds slow traffic, given that it rains at noon?" Using the conditional probability formula and adding over all the remaining variables.

When the end of the probability calculations, the one with the highest probability value chose, and the times are adjusted. Traffic events increase the calculated travel time: slow increases its time by 75%, relatively slow 50%, relatively fast 25%, and fast event it does not suffer any adjustment. If there is any rain level, the model adjusts the travel time by 25% for rain, 50% for heavy rain, and 75% for heavy rain.

Fig. 7. Tree diagram.

The model adjusts these parameters until obtaining results close to those desired, minimizing the error between both sets. This error measure calculates the discrepancy between the real output of the network and the desired output (Figure 8); the closer it is to zero, the better is the result.

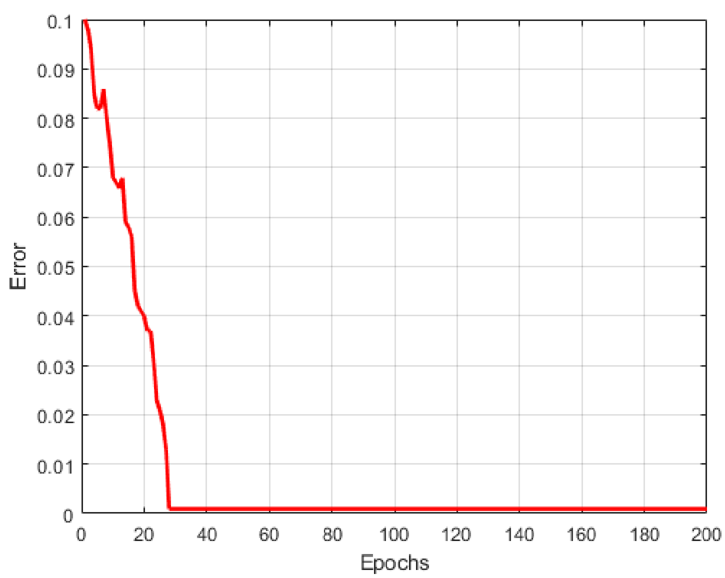


Fig. 8. Convergence to zero of the mean square error.

The proposed model approximates the Fuzzy output value with the sampled value of the traveled time. The model learns from an input vector with travel data, an output vector with times estimated by the model, 1000 epochs with a minimum error of 0.01, and a learning rate of 0.001.

Table 2 shows the times estimated with Fuzzy Logic compared with the times estimated applying the M-ANFIS model. The results of both models compare with the times obtained in the sampling. To reach the mean absolute percentage error of the Fuzzy Logic models and the mean fundamental percentage error of the M-ANFIS model. Table 3 shows the means of the FL percentage errors with 25.06% and 5.14%, the mean and variance of M-ANFIS, 11% and 0.14%, 46.75%, and 26.12%, respectively, for the probabilistic model, thus verifying that there is less variability with the M-ANFIS model.

Once the error calculates, the weights adjust, which modify the model's membership functions, below show the difference of the membership functions offered before learning and after, see Figure 9(a) and 9(b). As in the study case, there were no rains, rules of this variable were not activated, and did no modify them in the investigation. Respectively for the probabilistic model, thus verifying that there is less variability with the M-ANFIS model.

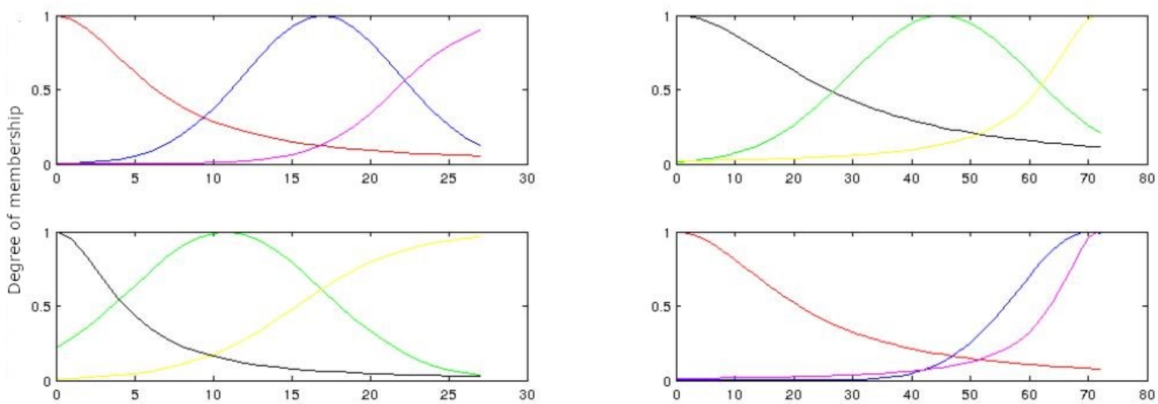
Table 2. Fuzzy output and Probabilistic model comparison with adaptive output

	Fuzzy variable			Real valor (min)	Fuzzy Logic		M-ANFIS		Probabilistic model	
	P	E	Z		Output	Difference	Output	Difference	Output	Difference
1	0	1	1	3.3	2.9	0.4	3.2213	0.0787	2.61	0.69
2	0	7	1	2.6	2.552	0.048	2.706	0.106	2.16	0.44
3	0	1	21	0.25	0.2625	0.0125	0.263	0.013	0.8224	0.5724
4	0	1	21	1.1	0.57	0.53	1.128	0.028	0.801	0.299
5	0	1	1	1.67	1.695	0.025	1.699	0.029	1.29	0.38
6	0	1	1	0.48	1.1078	0.6278	0.487	0.007	1.91	1.43
7	0	1	1	1.37	0.87	0.5	1.213	0.157	1.5	0.13
8	0	1	1	1.2	1.088	0.112	1.073	0.127	0.88	0.32
9	0	1	1	1.7	1.856	0.156	1.818	0.118	0.5	1.2
10	0	1	1	2.1	1.5296	0.5704	2.16	0.06	2.5	0.4
11	0	1	42	4	3.977	0.023	3.98	0.02	3.25	0.75
12	0	23	21	2.9	3.2038	0.3038	3	0.1	2.12	0.78
13	0	14	1	3.3	4.8361	1.5361	3.49	0.19	2.98	0.32
14	0	1	21	1	1.1625	0.1625	0.8715	0.1285	0.256	0.744
15	0	1	21	1.5	1.8075	0.3075	1.579	0.079	0.587	0.913
Total				28.47	29.4178		28.6888		24.1664	

Table 3. Mean absolute percentage vs error

	Sampled travel time	Time with FL	Error percentage with FL	Time with M-ANFIS	Error percentage with M-ANFIS	Time with the Probabilistic model	Error percentage with stochastic model
1	3.3	2.9	12.1%	3.2213	2.4%	2.61	20.9%
2	2.6	2.552	1.8%	2.706	4.1%	2.16	16.9%
3	0.25	0.2625	5.0%	0.263	5.2%	0.45	80.0%
4	1.1	0.57	48.2%	1.128	2.5%	0.801	27.2%
5	1.67	1.695	1.5%	1.699	1.7%	1.29	22.8%
6	0.48	1.1078	130.8%	0.487	1.5%	1.2	150.0%
7	1.37	0.87	36.5%	1.213	11.5%	1.5	9.5%

8	1.2	1.088	9.3%	1.073	10.6%	0.88	26.7%
9	1.7	1.856	9.2%	1.818	6.9%	0.5	70.6%
10	2.1	1.5296	27.2%	2.16	2.9%	2.5	19.0%
11	4	3.977	0.6%	3.98	0.5%	3.25	18.8%
12	2.9	3.2038	10.5%	3	3.4%	2.12	26.9%
13	3.3	4.8361	46.5%	3.49	5.8%	2.98	9.7%
14	1	1.1625	16.3%	0.8715	12.9%	0.256	74.4%
15	1.5	1.8075	20.5%	1.579	5.3%	0.587	60.9%
Mean			25.1%		5.14%		46.75%
Variance			11.0%		0.14%		26.12%



(a) State of the streets (b) Traffic Zones

Fig. 9. Membership functions before and after learning.

5 Conclusions

In conclusion, the combination of adaptive systems and their quality of parameter adjustment with the knowledge of experts in the form "**if ... then**" rules, given a set of input and output data pairs, adapts and approximates the values to values close to the real ones. These systems are suitable for repeating the behavior of a nonlinear system whose mathematical model does not quickly obtain.

Chose the Fuzzy variable used in this work for their frequency and their occurrence, the study found more variables that occasionally affect traffic and that contain an impact on travel time; for future studies, it recommends to evaluate these factors and integrate them into a system that assesses these and other variables considered in different settings.

Optionally, it recommends comparing the results of the M-ANFIS system with other adaptability systems such as genetic algorithms and evaluate the efficiency that one of the two would have in a computer system; unlike ANNs, Genetic Algorithms use in the resolution of search and optimization problems genetically reproducing a population over a series of generations.

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