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International Journal of Combinatorial Optimization Problems and Informatics, 17(2), 2026, 451-470. ISSN: 2007-1558.
https://doi.org/10.61467/2007.1558.2026.v17i2.1208

Conceptual synthesis of mechanisms based on qualitative reasoning

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Abstract. This paper presents an innovative method for qualitative reasoning in the conceptual synthesis of mechanisms, leveraging AI-derived knowledge-based principles. The methodology allows for discretizing a mechanism's overall behavior without computational implementation. Relative motion between components, represented as qualitative states, is captured in a qualitative motion vector. These vectors form a general movement matrix that characterizes the mechanism's behavior, providing insights into component functions, movements, and transitions. By using ratchets as restriction functions, underlying behaviors are isolated from the matrix. These behaviors are used to generate conceptual designs for new mechanisms fulfilling specific kinematic functions. A case study is presented: synthesizing a mechanism that converts oscillatory rotation into unidirectional rotation using a differential gear train. The qualitative behavior of the resulting design is visualized in a vector diagram and compared with a Computer-Aided Design (CAD) simulation. This method provides a knowledge base for training AI models in conceptual synthesis without needing specialized software.
Keywords: Qualitative reasoning; conceptual design; function-structure-behavior; envisionment; ratchet.

Article Info
Received August 20, 2025
Accepted October 09, 2025

List of abbreviations

Abbreviations	Expansion
FSB	Function Structure Behavior
GMM	General Movement Matrix
QS	Quantity Space
PSQ	Positive Quantity Space
NQS	Negative Quantity Space
GQS	General Quantity Space
BB	Building Block
MTM	Motion Transformation Matrix
MV	Movement vector
ei	Qualitative state, where $0 \leq i \leq \infty$

1 Introduction

The conceptual design phase is a critical stage in mechanical design, during which the general characteristics that a device must possess to fulfill a specific function are defined (Taked et al., 1990). The success in this early stage depends mainly on the designer's experience, who can employ various techniques to achieve the task. In the literature (Gero, 1990; Rinderle, 1987; Rinderle & Hoover, 1990) it is suggested that the quality and performance of a design are directly influenced by the designer's ability to translate design requirements to functions. These functions are then transformed into physical structures or components that are integrated into the final concept, ensuring the desired behavior. This process is commonly known as the function-structure-behavior (**FSB**) relationship.

The level of abstraction and the method of expressing requirements as functions vary depending on the approach (Faltings & Sun, 1996; Gero, 1990). Different design objectives require different types of knowledge and innovation, which means that the techniques employed will not be the same in every case. In prototype-based design, designers can refer to mechanism's summaries (Grafstein and Schwarz, 1971; Hiscox, 2007) that detail mechanisms capable of performing a specific function, which may only need refinement or adaptation. In such cases, the conversion of a requirement into an FSB relationship is straightforward. However, this approach is only applicable when an appropriate mechanism already exists for the specific function, and the design requirements align well with a manual's guidance. For situations where the conversion of design requirements into an **FSB** relationship is not direct, case-based reasoning approaches can be used to develop new conceptual designs based on previous concepts.

In the early 1980s, many of these approaches were grounded in qualitative reasoning, a field of knowledge applied to conceptual design that involves creating qualitative descriptions of the behavior of simple mechanical elements. These elements are abstracted into specific concepts related to their shape, geometry, or structure and are then integrated using methods that can generate conceptual designs for complex devices. This process can start with entirely new mechanisms or from previous mechanisms which meet specific design requirements. Since the analysis in these approaches is done from the specific to the general to explain the overall behavior of the final mechanism, it is essential to have a library that describes the particular behavior of the underlying mechanisms that make it up. If such a library is not available, it must be created. This necessity makes most of these approaches computationally intensive due to the vast number of libraries that must meet design requirements and the enormous range of potential solutions. Additionally, each approach is implemented within a specific software intrinsically linked to its methodology. Moreover, each approach is embedded within a specific software framework that is inherently tied to its underlying methodology. This often necessitates the use of complex algorithms and specialized algebraic rules, which in turn demand supplementary quantitative methods, such as configuration space (**CS**), to describe the final concept and its behavior. As a result, the emphasis has increasingly shifted toward non-qualitative methods, leading to a decline in research focused on the application of qualitative reasoning in conceptual design.

In this paper, we introduce a non-computational method called inverse envisioning to synthesize a new mechanism based on an existing mechanism given some design requirements. This approach involves principles of qualitative reasoning for discretizing the general behavior or envisionment of a mechanism using basic numerical operators and inequalities, which are represented in a general movement matrix (**GMM**) and illustrated in a vector diagram. By analyzing this matrix and applying ratchets as constraint functions, we can isolate specific underlying behaviors or qualitative states (referred to as qualitative behavior) that, collectively, fulfill the design requirements. The main advantage of this method is that it provides a straightforward, standardized qualitative analysis that narrows down the number of feasible solutions based on the mechanism's inherent properties. This approach offers insight into the overall behavior of a given mechanism and its underlying kinematics. Once a mechanism has been analyzed from a structural perspective, it becomes possible to accurately determine the type of movement it generates and identify the design options that fulfill specific requirements. These insights can then be leveraged to develop new conceptual designs, which may serve as foundational principles for applying artificial intelligence in the conceptual design of mechanisms.

2 Background

2.1 Qualitative reasoning basic principles

Qualitative reasoning in the conceptual synthesis of mechanisms was once a significant area of research but has since been largely abandoned. Early studies primarily focused on developing qualitative computational simulations to describe and predict the general behavior of physical systems, with the goal of establishing foundational principles for artificial intelligence. Forbus (1980, 1981, 1984) modeled systems such as boilers and oscillators to explore the qualitative motion of rigid bodies, like balls, by analyzing the relationship between an object and the space it occupies, its contact with surfaces, and the trajectory it follows under the influence of an external force described qualitatively. The general behavior of the system, referred to as *envisionment*, was derived through a process of analysis and refinement known as *envisioning*. This behavior was divided into regions grouped by a finite, distinguishable set of similar characteristics, termed qualitative states. In the work of De Kleer & Brown (1984), the behavior of systems, such as a pressure regulator, was represented in a state diagram produced by a computer program called ENVISION. This program utilized qualitative differential equations known as confluences, establishing many of the algebraic rules that would be used in later methodologies and serving as a technique to mitigate the inherent loss of information in qualitative approaches. This effort to enhance the accuracy of qualitative approaches involved the extensive use of algebraic operators and specific rules to relate them to qualitative variables. Other approaches were also developed to achieve this goal, such as MINIMA (Williams, 1990) based on Macsyma ® software, which combined qualitative and quantitative mathematical rules, resulting in a hybrid method for modeling systems like fluids in containers.

When an object exhibits a certain behavior, the finite set of values assumed by the variable is called the quantity space (**QS**), which is divided into two intervals: positive quantity space (**PQS**) and negative quantity space (**NQS**), separated by zero (0). Trave-Massuyes et al. (2003) extended the range of each interval by subdividing it into three subspaces, resulting in **NL**, **NM**, and **NS** (large negative, medium negative, and small negative, respectively), for the negative interval, and **PS**, **PM**, and **PL** (small positive, medium positive, and large positive, respectively), for the positive interval, creating a total of six intervals. This subdivision made the **QS** more manageable by characterizing quantities with finer distinctions and providing more orders of magnitude for a given variable value. However, it also required the use of additional operators and more complex qualitative algebraic rules for effective handling. D'Amelio et al (2013) proposed a computational method to reduce ambiguities by identifying them using a conflict triangle and incorporating additional qualitative and quantitative information. Kim (1990) suggested using arithmetic operators (> , <, =) and coupled qualitative vectors to minimize ambiguities in the simulation of linkages, although this approach was limited to the analysis of relative inclinations and angular positions of the links.

The arithmetic and qualitative algebraic rules applied to the simulation of the behavior of the systems did not change significantly, so the later approaches focused on developing computational tools to generate these simulations. Following the research line established by Forbus (1984), the computational methods for simulating and analyzing a single body were extended by other researchers to analyze the behavior of simple mechanisms with fixed axes such as ratchets (Faltings, 1988, 1992) gears and recoil escapement, (Forbus et al., 1991; P. Nielsen, 1990; P. E. Nielsen, 1988) or a driver system (Joskowicz, 1990). To generate envisionment, the envisioning consisted on establishing the relation between the geometry of the object (shape features), its position with respect to other bodies and the contact involved between them. These relationships are established in a set of computational sentences (*behavior predicates*) and compiled in an algorithm and processed by a program. In these approaches, the envisionment generated by the program is captured in a diagram known as *vocabulary places* (Faltings, 1990; Forbus, 1980) which is a portion of the CS of the mechanism. This analysis was extended to generate a complete simulation of the general behavior of complex mechanisms such as a clock (Forbus et al., 1991) and linkage mechanisms (Kim, 1990), studying its kinematics and dynamic aspects from a qualitative point of view. The qualitative analysis of the behavior of mechanisms provided the opportunity to use envisionment as a tool to perform the *conceptual synthesis of mechanisms* in *case-based* approaches, by using a previously designed device that meet certain requirements and then modifying it to meet other requirements.

2.2 Approaches related to conceptual design based on qualitative reasoning

The theories applied to conceptual design were categorized by Subramanian & Wang (1995) into three main types: structural, behavioral, and functional. Each theory focuses on generating the structure based on general principles. Additionally, there are combinations of these theories, such as the approach proposed in this paper, which merges behavioral and functional theories, with an emphasis on qualitative reasoning. Based on this, we can classify qualitative conceptual design approaches into two main branches: the first focuses on analyzing and modifying the geometry of specific devices, while the second examines the relationship between structure and the underlying behavior of components, which is more closely associated with type and structural synthesis.

2.2.1. From shape to function

In the first branch, the use of computational tools is crucial for applying qualitative reasoning techniques, which are closely related to the use of **CS** to generate design concepts. These tools are employed to reconstruct the geometry of mechanism components, adapting them to fulfill new functions. This process is referred to by Faltings & Sun (1992, 1993a, 1993b) as the *causal inversion technique*, which has been applied in the design refinement of ratchets and gear trains (Faltings & Sun, 1996). The key idea behind this technique is as follows: once the base mechanism's shape and geometry are envisioned, its *vocabulary places* are interpreted. From this, the metric predicates are derived to define the necessary characteristics that a mechanism must possess to achieve a particular function. Subsequently, the *modification operators* are specified to address discrepancies between the original device's specifications and the new requirements, refining the geometry of the previous device to better align with the new design goals. This approach represents the reverse of the process used to determine behavior based on form. Joskowicz & Addanki (1988) applied this method to express the kinematic behavior of elementary mechanical components and redefine their shapes through extensive use of **CS**. This approach was further analyzed by Sacks & Joskowicz (2010) for generating kinematic designs of complex mechanisms involving mobile contact parts and fixed axes.

2.2.2. Function-structure-behavior relationship

In the second branch, computational approaches based on qualitative reasoning use a different perspective to generate conceptual designs. They seek to abstract the **FSB** relationship in libraries that contain solutions of standardized concepts that can be integrated to obtain conceptual designs, and qualitative algebra rules are used to define and analyze spatial and underlying relationships between components of designs. This idea was early exposed by Gero (1990), whose design specifications based on **FSB** are visible in a graphical representation. (Hoover & Rinderle, 1989) explored the use of underlying and unexpected behaviors of a mechanism to preserve them as functions and integrate them into a description of a conceptual design to satisfy a given requirement, working as a fusion of compositional and functional theories. Li et al. (1996) presented a computational method developed only with kinematic criteria, extendable to multiple-inputs (**MI**) and multiple-outputs (**MO**) movements by adapting new design specifications and enrich the libraries of standardized concepts from available mechanisms classified according to the criteria established by the author. Thus, different and more complex design alternatives can be generated to fulfill the desired function.

The works of Chakrabarti & Bligh (1994) consisted of using the so-called "functional reasoning" to extract the domain of corresponding knowledge, necessary to generate the representations of the structures used, such as directions, lengths, orientation, torque and force, in qualitative terms. Subsequently, these qualitative representations are combined to generate different concepts of structures with single-input (**SI**) or **MI** and single-output (**SO**) or **MO** combined on single-input single-output (**SISO)**, single-input multiple-outputs **SIMO**, multiple-inputs multiple-outputs (**MIMO)** systems which fulfill a design requirement of a mechanism like a mechanical transmission (Chakrabarti & Bligh, 1996a, 2001). These constraints can be propagated according to the procedure, to generate spatial representations of the previous concepts (Chakrabarti & Bligh, 1996b) using the ART® package, which satisfies the same solution. It is important to highlight that these works, building on the approaches of Forbus, Faltings, and Nielsen, mark a transition to a new AI paradigm that focuses on generating concepts through the use of sign algebra and building blocks. In this regard, in the works of Chiou (1994), Chiou and Sridhar (1999) and Kota & Cuiou (1992), the behavior of a device is abstracted at its most basic form-function levels in concepts known as primitive mechanisms or building blocks (**BB**), which perform a certain transformation of movement. For instance, in the design of a device whose design requirement is to convert a horizontal rotation (input) into a perpendicular rotation (output), the structures responsible for performing this function can be two conical gears that change the direction of the rotation. The joint function of both components expressed as BB can be abstracted to a single structure to form a mechanism of greater complexity that describes this behavior; likewise, components such as a cam and its follower or an articulated linkage, a rack and a pinion can be represented by a building block that performs a specific function. The set of **BBs** can be assembled in different levels of abstraction by an ADAMS computer program. The final concept is embodied in a motion transformation matrix (**MTM**) that provides the information of the final configuration of the mechanism. This methodology is also taken to an approach combining dual algebra to separate the topological function of the mechanism kinematics (Moon & Kota, 2002), or to perform synthesis of compliant mechanisms (Kim, 2005).

A variant of the BBs method, which focuses on reusing concepts from previous mechanisms, is the approach developed by Han & Lee (2006). Its main feature is the abstraction of the function to be performed by the primary or basic mechanisms into a concept called the virtual function generator (**VFG**). This **VFG** forms a conceptual design, which is described using both a *graph structure* and *a kinematic diagram*. The approach builds on earlier work by Han & Lee (2002), in which 3x3 matrices, known as spatial configuration matrices (**SCM),** are generated according to rules dictated by qualitative algebra. These matrices represent the spatial configuration of a primitive mechanism's behavior based on the eight qualitative translational directions described by Nielsen (1988). Similar to **MTM** described by Kota & Chiou (1992). These **SCMs** are used to construct a matrix called the spatial configuration state matrix (**SCSM**), which describes the overall spatial configuration of complex mechanisms. The **SCSM** provides information about the initial, intermediate, or final qualitative states that a mechanism can assume. This is valuable for generating alternative conceptual designs that satisfy specific solutions defined by spatial requirements. The information derived from the various qualitative states of each primitive mechanism enables the creation of multiple design solutions.

An analogous reasoning is used by Feng et al. (2009) in a tipper tuck mechanism, in which the description of position and kinematic function of a primary mechanism is described by a qualitative information vector (**QIV**) and a position vector (**PV**) respectively.

These concepts obtained through a composition process dictated by the rules of qualitative algebra, are processed in a computational algorithm to generate the mechanical configuration matrix (**MCM**) and the output position vector (**OPV**) that describe the qualitative characteristics of the final conceptual design, which is finally analyzed dynamically in quantitative terms.

3 Development

3.1 Qualitative motion and direction

In qualitative analysis, the movement of rigid bodies is initially described by determining the qualitative directions of movement. The set of these movements is referred to as qualitative behavior, or simply behavior. Following the approaches of Forbus (1980, 1981, 1984), Nielsen (1990, 1988), and Kim (1990), the qualitative direction of movement is represented by the direction of a vector in space within a Cartesian coordinate system. The origin of this vector is positioned at a point of interest, either inside or outside the mechanism or one of its components. In this paper, a mechanism can have multiple independent frames of reference, and there is no distinction between absolute and relative frames. If a frame's origin is located on a moving part or a rotating axis, the frame will move or rotate with the part. In this case, the movement of the object relative to that frame will be considered absolute, even though the frame itself is mobile. A frame will be used as a reference solely to define input movement.

In the absence of an absolute magnitude for the qualitative directions of an object's movement, these directions are represented using signs. For translation, the $+$ sign indicates a positive direction, the $-$ sign represents a negative direction, and 0 signifies a position at the center of the reference frame, whether relative or absolute. For rotation, a $+$ sign corresponds to a clockwise rotation around an axis, a $-$ sign denotes a counterclockwise rotation, and 0 indicates no rotation. This use of signs differs from that employed by De Kleer & Brown (1984), Williams (1990), Han & Lee (2002) and Trave-Massuyes et al. (2003), where signs are used to determine the value of a *qualitative variable*, representing changes (increase or decrease) in a process known as *confluence*. In their context, a $+$ sign indicates a departure of the variable from zero, and a $-$ sign indicates an approach toward zero, regardless of the side of the number line where the value resides. In this work, however, these signs will specifically be used to represent directions of movement.

3.2 Qualitative variables

According to De Kleer & Brown (1984), a qualitative variable is used to describe the behavior of a device, and it can take only a limited range of values. If the overall behavior of a system is determined by the individual behavior of its components, then a variable is defined to represent the behavior of each component. For instance, to describe the rotation of a gear around an axis "x", the variable ω_{1x} will be used.

As we stated previously, quantity space (**QS**) covers the finite set of values assumed by the variable, and naturally it takes two general intervals: positive and negative. The positive direction of a variable, such as the rotation in the previous example where $\omega_{1x} = +$, defines the interval within the qualitative space (**QS**) as the positive quantity space (**PQS**), with values in the domain $\omega_{1x} \in [0, \infty)$. If the variable has a negative direction, i.e., $\omega_{1x} = -$, the interval within the QS is defined as the negative quantity space (**NQS**), with values in the domain $\omega_{1x} \in (-\infty, 0]$. When the variable has a zero value, $\omega_{1x} = 0$, it indicates a state of no movement. This qualitative state can correspond to a transition between the **PQS** and the **NQS**, or vice versa.
The overall quantity Space (**QS**) of a piece or mechanism can be defined including all the qualitative values that occupy their qualitative states. This referred to as the General Quantity Space (**GQS**). It can also be defined by intervals, and in the order in which the movement of the components occurs. For instance, if a mechanical element, whose qualitative variable describes a motion that goes from negative to rest, will have a **QS** $\{-,0\}$. However, this may only be part of a particular range of motion, and its general behavior describes the **QS** $\{-,0,+\}$, which indicates that the motion is from negative to positive, passing through zero, or absence of movement. If this movement is oscillating, returning to the initial state, then its **QS** $\{-,0,+,0,-\}$. It should be noted that in order to pass from a positive qualitative state to a negative one and vice versa, one has to go through an intermediate state of absence of movement: a transition.
To define the qualitative value of a variable, the following operators will be used: $>, =, <$, or, depending on the case, $\geq, =, \leq$. These operators represent inequalities or disambiguation between variables and qualitative states, as explained in the cases below.

3.3 Qualitative state and Transition

In this work, we will define the qualitative state as the behavior of a body or a mechanism (group of bodies) represented by a unique combination of qualitative values between variables and delimited by other qualitative states. Therefore, a qualitative state indicates the behavior comprised in a range of specific values in a certain direction and delimited by the operators previously explained. A transition between qualitative states will be determined by the change of value or sign of some of its variables.

For the rotation variable ω_{1x}, at an initial state of no motion, its qualitative magnitude in terms of angular velocity is $\omega_{1x} = 0$, and the qualitative state that presents this angular velocity will be labeled as $\boldsymbol{e1}$. If the rotation is clockwise, following the convention sign of the cross product, the variable will be defined as $\omega_{1x} = +$. We will label this state as $\boldsymbol{e2}$. The angular velocity that corresponds to the initial states will be represented as $e1: [\omega_{1x}]$ or as $\omega_{1x}: [\boldsymbol{e1}]$. The same notation applies to state $\boldsymbol{e2}$.

The distinction between both states can be represented by the relation $\omega_{1x}: [\boldsymbol{e2}] > \omega_{1x}: [\boldsymbol{e1}]$. We can represent this in a more compact way like $\omega_{1x}: [\boldsymbol{e2} > \boldsymbol{e1}]$. If a gear has a magnitude of 10 in state $\boldsymbol{e2}$, and in a subsequent arbitrary state $\boldsymbol{ei}$ it has a value of $\omega_{1x} = -10$, the relationship between the two states is expressed as $e2: [\omega_{1x}] = |ei: [\omega_{1x}]|$, which can also be represented as: $\omega_{1x}: [\boldsymbol{e2} = -|\boldsymbol{ei}|]$. Both qualitative states are located in a different **QS**, so to move from state $\boldsymbol{e2}$ to the random state $\boldsymbol{ei}$, the variable ω_{1x} must pass through 0 to experience a logical transition. This change of states and its transition of this variable will be represented as ω_{1x}: **QS**$\{+,0,-\}$.

On the other hand, if another gear of the same mechanism rotates around a parallel axis with an angular velocity ω_{2x} in $\boldsymbol{e2}$ and the value of its qualitative magnitude is identical to $\boldsymbol{e2}: [\omega_{1x}]$, i.e. 10, then it can be established that $\omega_{1x}: [\boldsymbol{e2}] = \omega_{2x}: [\boldsymbol{e2}]$, or: $\boldsymbol{e2}: [\omega_{2x} = \omega_{1x}]$.

According to the approaches of De Kleer & Brown (1984), and Williams (1990) and Trave-Massuyes et al.(2003), the operators can only be used in the variables and not in the confluences, and an equality $(=)$ between two qualitative states indicates a relation of proportionality between the change of two confluences (for example, when comparing two variables "y" and "z" that changed proportionally, the confluences are $\partial y = \partial z$). However, this do not imply that their numerical values are identical; it only indicates the direction of the change of the variable in its **QS** for a certain qualitative state. Therefore, the only absolute value that could be assigned to a variable when using the equal sign is 0.
In this work, operators and signs will be applied to a variable to simplify the analysis. The aim of using these operators is to minimize the geometric and numerical information required. Their application follows a similar approach to that of Kim (1990). The equal sign will be used to denote a specific value in the following cases:

- To indicate that a variable is zero or has a direction in the **PQS** or **NQS**.
- To indicate that two different variables have the same value in the same qualitative state.
- To indicate that two qualitative states of the same variable have the same value.

The representation of a qualitative state will be made using a motion vector (**MV**), which is constructed by specifying the signs of the qualitative variables in the order of their definition for a given state. This is followed by an inequality to clarify the values of the variables that require disambiguation. An example of how to build an **MV** will be seen later in section 4.3.2. Also, the definition of envisionment will be used to represent the general behavior of the mechanism. The set of vectors that define the envisionment will be known as general movement matrix **(GMM)** which also provides information of the spatial configuration of the mechanism as **MTMs**. The process to build the **GMM** will be defined as envisioning as well, and it will not be done with a computational approach, but with a simple, compact and extensible reasoning. The graphic representation of the **GMM** will be made through a vector diagram.

3.4 Ratchet

A ratchet is a mechanism that allows the rotation of an element such as a shaft, in one direction while restricting it in the opposite direction. According to Chiou (1994), a ratchet can be viewed as a restriction function that connects one or more **BBs** and can be mathematically represented by an inequality, which operates under the following two cases:
Consider a system of an axis and a frame that is considered immobile. The rotation of the axis will be allowed in one direction (positive or negative, as required). For the present example, the allowed direction is $+$. Thus, if the angular velocity of an element A is ω_{Ax} , its inequality will be the Equation (1):

$$\omega_{Ax} \geq 0 \tag{1}$$

In this case, the use of a ratchet as a restriction function allows isolating an entire **GQS** range, either the **PQS** or the **NQS.** For the example, the chosen one is **PQS.**

The second case arises when the ratchet is positioned between two axes that exhibit relative motion with respect to each other. In this case an element A that rotates around an axis "x" with an angular velocity ω_{Ax}, and an element B that rotates around the same axis or concentric to it, with an angular velocity ω_{Bx}, they will have a movement restriction such that, if we consider B as the 'mobile frame', the resulting inequality will be given by Equation (2):

$$\omega_{Ax} \geq \omega_{Bx} \tag{2}$$

In this case, using a ratchet as a restriction function enables the isolation of a portion of the **GQS**, or even a specific interval within the **PQS** or **NQS**. This is crucial, as the inequality implies that if element B can rotate with either a positive or negative sign, link A must rotate with the same or greater angular velocity (regardless of whether it is positive or negative). This also means that link A cannot rotate at a lower angular velocity than link B, making B function as a mobile frame. For example, if $\omega_{Ax} = -10\ s^{-1}$ and $\omega_{Bx} = -11\ s^{-1}$, then they are valid values, however the values $\omega_{Ax} = -12\ s^{-1}$ and $\omega_{Bx} = -11\ s^{-1}$ will not be valid.

3.5 Methodology for conceptual design

The definitions stated above will be applied in a methodology that allows obtaining conceptual design of a new mechanism based on a previous mechanism. The methodology followed in this work consists of the next points:

I. To determine the design requirements for a mechanism based on the functional-structural-behavior (**FSB**) relationship between its input and output, specifically focusing on the functional movement derived from its structure.

II. To select a mechanism whose **FSB** description, based on its input and output, aligns with the specified design requirements.

III. Generate the **GMM** according to the following steps:
   a) To identify the input motion element, define the reference axes and sign conventions, and describe the movement variables for any mechanical element, assuming a general case of input motion.
   b) To generate a description of the general behavior based on the **FSB** relationship. It should be assumed that the mechanism produces a movement that meets its qualitative description in terms of the relationship between its input and output, in the required directions, and for the necessary number of cycles. This movement must be transmitted to each link in a logical sequence, addressing all possible movement cases for each link, as long as they are feasible. Then, the **MV** for each one of the qualitative states is generated, as well as the necessary disambiguation among the qualitative variables (see section 4.3.2). Since this is a qualitative analysis, the designer must possess logical reasoning skills to understand the effects of the movement of bodies in space, their interactions with other bodies, and the prior functioning of mechanisms. This knowledge is essential to effectively envision and select the appropriate mechanism as a basis.
   c) To build the **GMM**. First, organize the qualitative states, generate a description of their QS according to the established convention, and clarify any ambiguities between related variables (see Table 1).

IV. Implement inverse envisioning: analyze the **GMM** and isolate the required movements based on the description in the case study, using ratchets as restriction functions. Begin by studying the relationships between input and output variables, then define their properties, including transitions, periodicity, maximum and minimum values, and qualitative symmetries. Finally, establish the relationships between unlinked variables of the intermediate links.

V. Generate the concept: Using ratchets as restriction functions, isolate the **GMM** that meets the design requirements based on its FSB movement description. Identify any alternative solutions derived from the analysis and create a sketch, diagram, or visual representation of the new device.

VI. Generate the new envisioning: Using the remaining qualitative states, create a new **GMM** and vector diagram that describe the behavior of the new mechanism.

VII. Archive the GMM obtained in Step III, along with the one representing the envisioning of the new mechanism from Step VI. This information can be used later in a case-based analysis to develop more complex mechanisms derived from the current ones. This last aspect is not included in this paper.

VIII. Simulate the mechanism's movement. Compare its behavior with the predictions made in the new envisioning to verify alignment.

IX. Build new mechanism. This step is not included in the present study.

4 Case study

The methodology followed in this work focuses on obtaining information about the general behavior of pre-existing mechanisms, where the spatial configuration is already defined. Once the general behavior is determined, the qualitative useful states of the envisioning will be analyzed and refined using the methodology presented in the previous section. For the case study, the conceptual synthesis of a mechanism than converts oscillatory rotation into unidirectional rotation is developed.

4.1 Design requirements.

The required mechanism should convert an oscillatory rotation of an input link (either positive or negative) into unidirectional rotation in the output link (either positive or negative) concentrically. The design requirements must be expressed in terms of its qualitative behavior. Therefore, the input rotation will be expressed as ω_{inx} and the output rotation as ω_{outx}, So, the quantity space of the input and the output is ω_{inx}: $\mathbf{QS}\{-,0,+\}$ and ω_{outx}: $\mathbf{QS}\{-,0]$ or ω_{outx}: $\mathbf{QS}[0,+\}$, respectively.
It is important to note that for the output variable, the output rotation is considered only in the positive direction, thus the PQS is accounted for. In the literature, it can be found various mechanisms that convert an oscillatory input rotation—such as from a gear, pulley, or shaft—into continuous rotary motion. This transformation is achieved by employing ratchets in conjunction with a train of bevel gears, which then transmit the converted rotary motion to a mechanical output element. Depending on the design, these devices transmit the output motion either perpendicularly (Grafstein & Schwarz, 1971; Hiscox, 2007) or concentrically (Kolokolnikov, 2011) relative to the input motion. One application of these mechanisms is in the conversion of rotation in strapping machines (Nix, 1992) or in the oscillatory pedaling motion used in certain bicycle prototypes (Kramer & Desmond, 2014; Salueña, 2007; Weber, 2014)

4.2 A mechanism that meets the design requirements

The base mechanism analyzed is a differential gear train, as shown in Fig. 1. The system consists of conical gears, main input and output gears, and planetary gears mounted on a carrier (crown). Power is supplied to the mechanism via the pinion drive, which distributes the energy throughout the system, ultimately transmitting it through both output shaft-gears. The difference in rotation between the left and right outputs allows for one to rotate in the negative direction while the other rotates in the positive direction. Additionally, other types of differential movement are possible, such as both outputs rotating in the same direction (either positive or negative) at the same or different angular velocities, or one wheel rotating in any direction while the other remains stationary. In all cases, a net rotational motion is maintained between the two, originating from an axis. The behavior of these two links (left and right output gears/shafts) can be qualitatively described as $\mathbf{QS}\{+,0,-\}$, a domain that accommodates the $\mathbf{QS}$ outlined in the design requirements. This requires isolating only the portion of the $\mathbf{PQS}$ relevant to the output link. This feature enables the isolation of the design requirements within the context of our methodology, allowing for a modification to the mechanism. As shown in Fig. 2, the pinion is removed, transforming the crown from an input link to a carrier for the planetary gears. Additionally, the planetary gears are arranged in a crosshead configuration. This modification allows one of the left or right gears to serve as the output link, while the other functions as the input link.

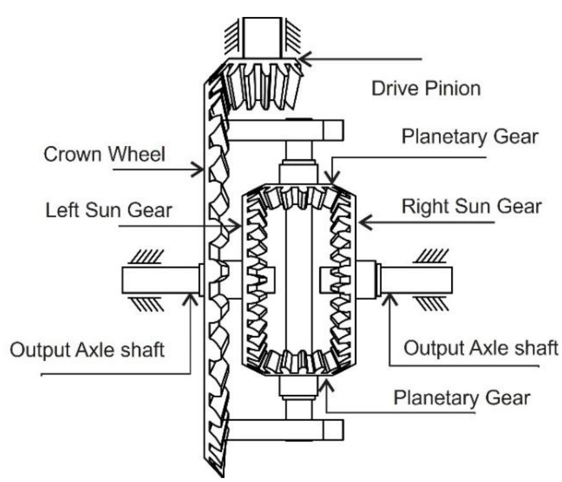

Fig. 1. Ordinary differential gear train.

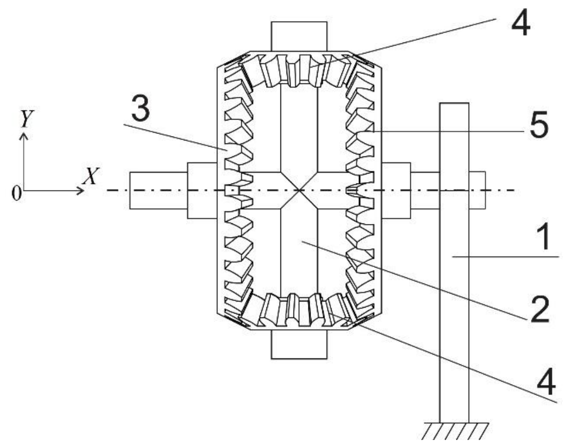

Fig. 2. Differential gear train modified for case study. It consists of i links and j axes (the "x" and "y" axes, located on the arms of link 2). In this configuration, link 1 serves as the frame, link 2 acts as the crosshead-shaped carrier, link 3 is a conical gear that functions as the input link, link 4 is a planetary bevel gear, and link 5 is a conical gear that serves as the output link.

4.3 Generate the GMM

4.3.1 Establish axes, sign convention and describe movement variables

As for the reference frames of the components shown in Fig. 2, it is assumed that frame 1 is stationary. Link 2 is connected to frame 1 and gears 5 and 3 are attached to the *x-axis* of link 2, which is considered mobile because it rotates around the *x-axis*. As a result, these gears do not have a stationary frame. Gear 4 (the planetary gear) is mounted on the *y-axis* of link 2, making it part of the mobile frame, and it does not make direct contact with the stationary frame 1.

In accordance with the design requirements for this case, the qualitative variables should describe the rotation of the links designated as input (3) and output (5). Thus, the variable analyzed is the angular velocity ω. In a qualitative analysis like this one, the goal is to describe rotation in terms of a specific qualitative state. For this reason, we use the variable ω_{ij}, where i represents the link number and j represents the axis. So, the input rotation variable ω_{inx} and the output rotation variable ω_{outx} will be represented by ω_{3x} and ω_{5x}, respectively. The variables ω_{2x} and ω_{4y} will represent the rotation of the crosshead (carrier) and the planetary gears around the *x-axis* and *y-axis*, respectively. For the pinion, x_p and y_p are the relative axes and O_p is their common relative origin. For the analysis, the sign convention of movements shown in Fig. 3, is adopted.

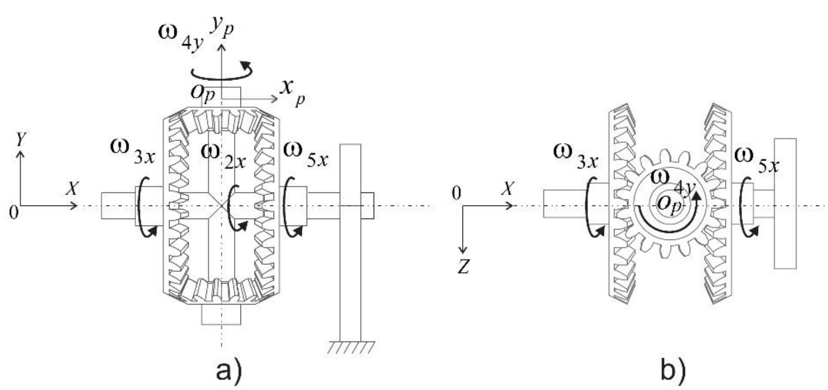

Fig. 3. Convention of signs to define the movement of the links of the mechanism: (a) front view and (b) top view of the mechanism. Positive rotation of links is shown.

4.3.2 Generation of the motion vectors

To generate qualitative motion vectors, we assume the system starts from rest, with an initial qualitative state of $\boldsymbol{e0}$, where the net movement of each component of the mechanism is zero. Then, the cases when the input link has a **PQS** $\omega_{3x} > 0$, and when it has a **NQS** $\omega_{3x} < 0$, are analyzed, representing a movement that encompasses an oscillatory rotation that passes through both **QS**. The movement logic involves starting with the input link rotating in a positive direction until it reaches a maximum. It then returns to zero, shifts to a negative direction, and finally returns to the initial state of rest.

The mechanically possible movements of the other components, derived from both general movement cases, will be analyzed. Invalid states will be excluded from the matrix and, consequently, not considered in the analysis. For example, if the input gear rotates positively while the fixed carrier 2 remains stationary, this will cause the planetary gear to rotate positive, resulting in a negative rotation of output link 5. An invalid movement in this case would be a positive rotation of output gear 5, which should be excluded from the matrix. Ultimately, all the resulting qualitative states will be organized in a logical sequence, reflecting the intended operation (envisionment) of the mechanism.

The motion vectors are represented in each of the rows of the **GMM**. The variables will be organized as follows: ω_{4y}, ω_{3x}, ω_{2x} and ω_{5x} (see Table 1). The variables that have a common axis are ω_{3x}, ω_{2x} and ω_{5x}, so the disambiguation applied to these variables is organized in the last column of the **GMM**. A vector representing a qualitative state will be defined by:

$$\boldsymbol{ei}: \omega_{4y}, \omega_{3x}, \omega_{2x}, \omega_{5x} \quad \textbf{(3)}$$

To disambiguate one qualitative state from another, the disambiguation will follow the same order of variables, relating them either to each other or to the value zero (0), using the appropriate operators ($<, =, >$). As an illustration of the procedure followed in this work to establish the **GMM**, the motion analysis and **MV** of the qualitative states $\boldsymbol{e0}, \boldsymbol{e1}, \boldsymbol{e2}, \boldsymbol{e3}$ and $\boldsymbol{e4}$ will be developed in the next sections.

Qualitative state $\boldsymbol{e0}$ *(repose):*

In this state, which is represented in Fig. 4, the mechanism remains stationary. As a result, no component exhibits angular velocity. The qualitative motion vector (**MV**) for this state, based on the variable order established in Equation **Error! Reference source not found.**, will be: $\boldsymbol{e0}$: 0,0,0,0; $\omega_{3x} = \omega_{2x} = \omega_{5x}$.

The disambiguation to the right of the vector is used to specify the relationships between the values of the variables.

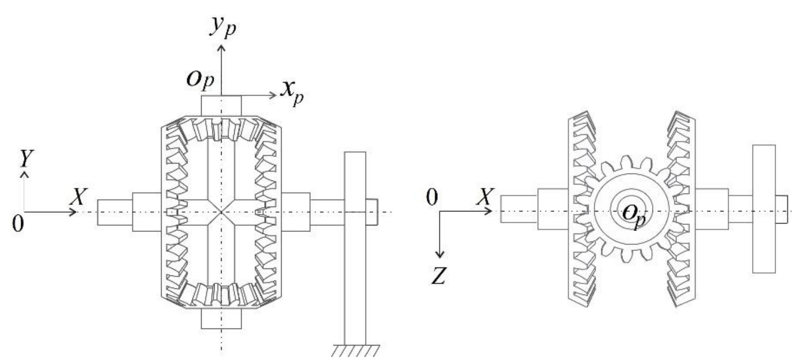

Fig. 4. Qualitative state $e0$ of a differential gear train.

Qualitative state $\boldsymbol{e1}$*:*

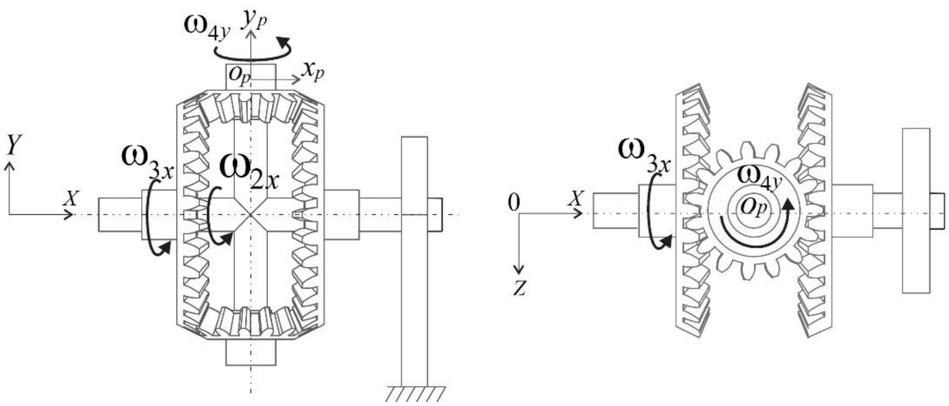

Fig. 5. Qualitative state $\boldsymbol{e1}$ of a differential gear train.

In this state, represented in Fig. 5, gear 3 begins to rotate positively, causing its angular velocity to be $\omega_{3x} = +$. This motion drives carrier 2, giving it an angular velocity of $\omega_{2x} = +$. For this to occur, the planetary gear must rotate with a positive angular velocity of $\omega_{4y} = +$. At this instant, gear 5 remains stationary, with an angular velocity of $\omega_{5x} = 0$. Therefore, the resulting **MV** is: $\boldsymbol{e1}: +, +, +, 0;\ \omega_{3x} > \omega_{2x} > \omega_{5x}$

The disambiguation to the right of the vector indicates that, up to this point, gear 3 has the highest angular velocity magnitude.

Qualitative state $\boldsymbol{e2}$*:*

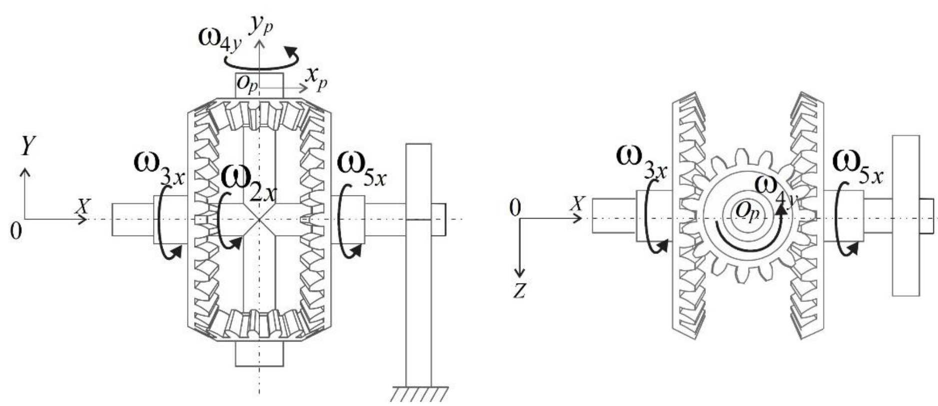

Fig. 6. Qualitative state $\boldsymbol{e2}$ for a differential gear train

In this state (Fig. 6), gear 3 continues to rotate positively, with its angular velocity $\omega_{3x} = +$, driving carrier 2 and causing it to rotate with an angular velocity $\omega_{2x} = +$. For this to occur, the planetary gear must also rotate with a positive angular velocity $\omega_{4y} = +$. Additionally, gear 5 begins to rotate with an angular velocity $\omega_{5x} = +$. Thus, its **MV** is: $\boldsymbol{e2}: +, +, +, +;\ \omega_{3x} > \omega_{2x} > \omega_{5x}$

The disambiguation to the right of the vector indicates that, while all the variables are within the **PQS**, gear 3 has the highest angular velocity magnitude, followed by the carrier, and then the output gear.

Qualitative state $e3$*:*

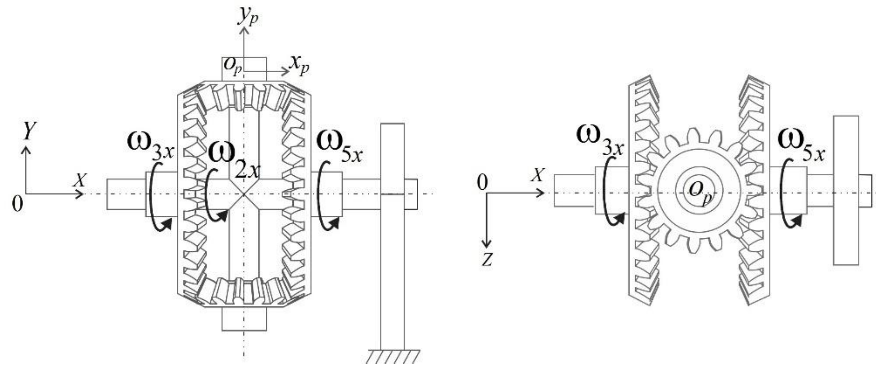

Fig. 7. Qualitative state $\boldsymbol{e3}$ for a differential gear train

In this state (Fig. 7), gear 3 rotates positively with an angular velocity of $\omega_{3x} = +$. Gear 5 also reaches an angular velocity of $\omega_{5x} = +$, equaling the magnitude of gear 3. For this to occur, the planetary gear must have zero angular velocity, $\omega_{4y} = 0$, which results in the carrier 2 rotating positively with an angular velocity of $\omega_{2x} = +$, equal in magnitude to that of gears 3 and 5. So, the **MV** of this state is: $\boldsymbol{e3}\!: \mathbf{0}, +, +, +;\ \omega_{3x} = \omega_{2x} = \omega_{5x}$

The disambiguation to the right of the vector indicates that all the variables are in the **PQS** and have the same angular velocity.

Qualitative state $\boldsymbol{e4}$

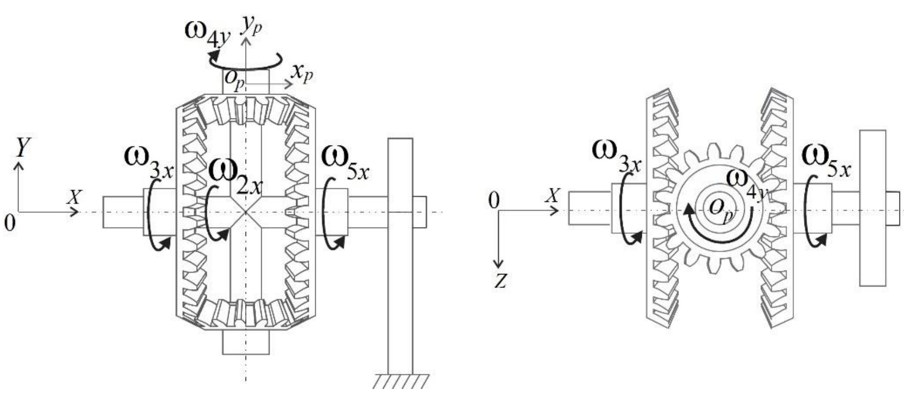

Fig. 8. Qualitative state $\boldsymbol{e4}$ for a differential gear train

In $\boldsymbol{e4}$ (Fig. 8), gear 3 rotates positively with an angular velocity of $\omega_{3x} = +$, while gear 5 continues to rotate with an angular velocity of $\omega_{5x} = +$, but with a magnitude greater than that of gear 3. For this to occur, the planetary gear must rotate with a negative angular velocity relative to its axis, $\omega_{4y} = -$, which causes carrier 2 to maintain a positive angular velocity, $\omega_{2x} = +$. The velocity of carrier 2 is smaller than that of gear 5 but greater than that of gear 3. The resulting motion vector (MV) is: $\boldsymbol{e4}\!: -, +, +, +;\ \omega_{3x} < \omega_{2x} < \omega_{5x}$.

The disambiguation to the right of the vector indicates that all the variables are within the PQS, but the relationship between the angular velocities has reversed, resulting in link 5 having the highest angular velocity.

Applying the same analysis to all the possible combinations of movements of the gear train, and following a logical movement of the links, the whole envisionment of the mechanism can be obtained. The **GMM** that includes all the **MV** of the mechanism is presented in Table 1. This matrix consists of 17 motion vectors (**MV**), with each vector containing 4 variables, defined in the order: ω_{4y}, ω_{3x}, ω_{2x}, ω_{5x}. The information within each vector represents a unique qualitative state. It can be observed that the qualitative state $\boldsymbol{e17}$ is identical to $\boldsymbol{e2}$, indicating that from this point, the cycle restarts. Therefore, the complete cycle, excluding the rest state $\boldsymbol{e0}$, consists of 16 states. For the analysis of the movement matrix, which focuses on states within a continuous cycle, states from $\boldsymbol{e1}$ to $\boldsymbol{e17}$ will be examined. It is important to note that a successive state can return to the previous one in reverse, ensuring the continuity of motion.

Table 1. General Movement Matrix

Qualitative State	Qualitative Variable				Disambiguation
	ω_{4y}	ω_{3x}	ω_{2x}	ω_{5x}	
$\boldsymbol{e0}$	0	0	0	0	$\omega_{3x} = \omega_{2x} = \omega_{5x}$
$\boldsymbol{e1}$	+	+	+	**0**	$\omega_{3x} > \omega_{2x} > \omega_{5x}$
$\boldsymbol{e2}$	+	+	+	+	$\omega_{3x} > \omega_{2x} > \omega_{5x}$
$\boldsymbol{e3}$	0	+	+	+	$\omega_{3x} = \omega_{2x} = \omega_{5x}$
$\boldsymbol{e4}$	-	+	+	+	$\omega_{3x} < \omega_{2x} < \omega_{5x}$
$\boldsymbol{e5}$	-	0	+	+	$\omega_{3x} < \omega_{2x} < \omega_{5x}$
$\boldsymbol{e6}$	-	-	+	+	$\omega_{3x} < \omega_{2x} < \omega_{5x}$
$\boldsymbol{e7}$	-	-	0	+	$\omega_{3x} < \omega_{2x} < \omega_{5x}, \lvert\omega_{3x}\rvert = \omega_{5x}$
$\boldsymbol{e8}$	-	-	-	+	$\omega_{3x} < \omega_{2x} < \omega_{5x}, \lvert\omega_{3x}\rvert > \omega_{5x}$
$\boldsymbol{e9}$	-	-	-	0	$\omega_{3x} < \omega_{2x} < \omega_{5x}, \lvert\omega_{3x}\rvert > \lvert\omega_{2x}\rvert$
$\boldsymbol{e10}$	-	-	-	-	$\omega_{3x} < \omega_{2x} < \omega_{5x}, \lvert\omega_{3x}\rvert > \lvert\omega_{2x}\rvert > \lvert\omega_{5x}\rvert$
$\boldsymbol{e11}$	0	-	-	-	$\omega_{3x} = \omega_{2x} = \omega_{5x}$
$\boldsymbol{e12}$	+	-	-	-	$\omega_{3x} > \omega_{2x} > \omega_{5x}, \lvert\omega_{3x}\rvert < \lvert\omega_{2x}\rvert < \lvert\omega_{5x}\rvert$
$\boldsymbol{e13}$	+	0	-	-	$\omega_{3x} > \omega_{2x} > \omega_{5x}, \lvert\omega_{3x}\rvert < \lvert\omega_{2x}\rvert < \lvert\omega_{5x}\rvert$
$\boldsymbol{e14}$	+	+	-	-	$\omega_{3x} > \omega_{2x} > \omega_{5x}, \omega_{3x} < \lvert\omega_{2x}\rvert < \lvert\omega_{5x}\rvert$
$\boldsymbol{e15}$	+	+	0	-	$\omega_{3x} > \omega_{2x} > \omega_{5x}, \omega_{3x} = \lvert\omega_{5x}\rvert$
$\boldsymbol{e16}$	+	+	+	-	$\omega_{3x} > \omega_{2x} > \omega_{5x}, \omega_{3x} > \lvert\omega_{5x}\rvert$
$\boldsymbol{e17}$	+	+	+	0	$\omega_{3x} > \omega_{2x} > \omega_{5x}$

4.4 Analysis of the GMM for Inverse envisioning

When a mechanism needs to be modified to change its function, several key questions arise: Which part of the mechanism needs to be altered? What underlying behaviors can be leveraged? What new behaviors will emerge as a result of these modifications? These questions are answered through the **GMM** analysis. The movement matrix is organized into a sequence of movements, where transitions between qualitative states for each variable follow a logical progression, moving from a **PQS** state to an **NQS**, or vice versa. These transitions are represented by **QS**$\{+,0,-\}$ or **QS**$\{-,0,+\}$. The order is determined by the movement of the input link (column 3). The variable ω_{4x} is placed in column 2 because its corresponding link is positioned on an axis that is neither parallel nor concentric to links 2, 3, and 5, meaning no disambiguation relationships are established between the variables ω_{3x}, ω_{2x}, ω_{5x} and ω_{4y}.

4.4.1 Transitions, periodicity, maximum and minimum values and qualitative symmetries

Each angular velocity variable has a total of 7 states in the **PQS**, 7 states in the **NQS**, and 2 states in the zero-movement condition. The maximum or minimum angular velocity values occur in the intermediate qualitative states of the **PQS** or **NQS**, respectively, for each variable. The states from $\boldsymbol{e1}$ to $\boldsymbol{e8}$ describe a behavior of the mechanism where the input gear transitions from rest to positive rotation, reaches its peak velocity in $\boldsymbol{e3}$, and then decrease to zero in $\boldsymbol{e5}$. From $\boldsymbol{e6}$, the input gear rotates negatively until it reaches a maximum negative value in the **NQS** at $\boldsymbol{e9}$. In this range of motion ($\boldsymbol{e1} - \boldsymbol{e8}$) the input link describes a **QS**$\{+,0,-\}$. On the other hand, when the output gear rotates positively, it describes a **QS**$\{0,+\}$ and returns to rest at $e9$. Upon completing the movement cycle, which spans from $\boldsymbol{e1}$ to $\boldsymbol{e16}$, link 3 follows a **GQS** of $\{+,0,-,0,+\}$, while link 5 follows a **GQS** of $\{0,+,0,-\}$. Thus, the maximum angular velocity of ω_{3x}occurs at $\boldsymbol{e1}$, with a transition from positive to negative at $\boldsymbol{e5}$ and from negative to positive at $\boldsymbol{e13}$. Its minimum value is reached at $\boldsymbol{e9}$. Table 2 presents the maximum and minimum values for all variables, along with their respective transitions.

Table 2. Maximum and minimum qualitative states and transitions.

Variable	$\boldsymbol{\omega_{4y}}$	$\boldsymbol{\omega_{3x}}$	$\boldsymbol{\omega_{2x}}$	$\boldsymbol{\omega_{5x}}$
Maximum	$\boldsymbol{e15}$	$\boldsymbol{e1}$	$\boldsymbol{e3}$	$\boldsymbol{e5}$
Minimum	$\boldsymbol{e7}$	$\boldsymbol{e9}$	$\boldsymbol{e11}$	$\boldsymbol{e13}$
Transition from **QSP** to **QSN**	$\boldsymbol{e3}$	$\boldsymbol{e5}$	$\boldsymbol{e7}$	$\boldsymbol{e9}$
Transition from **QSN** to **QSP**	$\boldsymbol{e11}$	$\boldsymbol{e13}$	$\boldsymbol{e15}$	$\boldsymbol{e1}$

The order of the qualitative states between maxima, minima, and their valid motion transitions generates the continuity of motion for each link. The maximum values are inferred by analyzing the relationship between transitions and the qualitative states in which the intermediate positions of a given variable lie within a specific **QS**. For example, the maximum value of ω_{5x} is conjectured to occur at $\boldsymbol{e5}$**,** as this state represents the intermediate position in the **PQS** for this variable, while the minimum value occurs at $\boldsymbol{e13}$, as it represents the intermediate position in the **NQS**. Thus, the relationship between the values of all the qualitative states of the same variable, considering its **GQS**, is as follows:

$$\omega_{4y}: [\boldsymbol{e1} > \boldsymbol{e2} > \boldsymbol{e3} > \boldsymbol{e4} > \boldsymbol{e5} > \boldsymbol{e6} > \boldsymbol{e7} < \boldsymbol{e8} < \boldsymbol{e9} < \boldsymbol{e10} < \boldsymbol{e11} < \boldsymbol{e12} < \boldsymbol{e13} < \boldsymbol{e14} < \boldsymbol{e15} > \boldsymbol{e16}]$$
$$\omega_{3x}: [\boldsymbol{e1} > \boldsymbol{e2} > \boldsymbol{e3} > \boldsymbol{e4} > \boldsymbol{e5} > \boldsymbol{e6} > \boldsymbol{e7} > \boldsymbol{e8} > \boldsymbol{e9} < \boldsymbol{e10} < \boldsymbol{e11} < \boldsymbol{e12} < \boldsymbol{e13} < \boldsymbol{e14} < \boldsymbol{e15} < \boldsymbol{e16}]$$
$$\omega_{2x}: [\boldsymbol{e1} < \boldsymbol{e2} < \boldsymbol{e3} > \boldsymbol{e4} > \boldsymbol{e5} > \boldsymbol{e6} > \boldsymbol{e7} > \boldsymbol{e8} > \boldsymbol{e9} > \boldsymbol{e10} > \boldsymbol{e11} < \boldsymbol{e12} < \boldsymbol{e13} < \boldsymbol{e14} < \boldsymbol{e15} < \boldsymbol{e16}]$$
$$\omega_{5x}: [\boldsymbol{e1} < \boldsymbol{e2} < \boldsymbol{e3} < \boldsymbol{e4} < \boldsymbol{e5} > \boldsymbol{e6} > \boldsymbol{e7} > \boldsymbol{e8} > \boldsymbol{e9} > \boldsymbol{e10} > \boldsymbol{e11} > \boldsymbol{e12} > \boldsymbol{e13} < \boldsymbol{e14} < \boldsymbol{e15} < \boldsymbol{e16}]$$

The analysis of the matrix reveals that the qualitative states from $\boldsymbol{e1}$ to $\boldsymbol{e8}$, which make up half of the envisioned cycle, exhibit a characteristic we will refer to as $qualitative\ symmetry$ with respect to states $\boldsymbol{e9}$ to $\boldsymbol{e16}$. This means that, for each variable, the absolute qualitative value of $\boldsymbol{e1}$ is equal to that of $\boldsymbol{e9}$, $\boldsymbol{e2}$ to $\boldsymbol{e10}$, $\boldsymbol{e3}$ to $\boldsymbol{e11}$, $\boldsymbol{e4}$ to $\boldsymbol{e12}$, $\boldsymbol{e5}$ to $\boldsymbol{e13}$, $\boldsymbol{e6}$ to $\boldsymbol{e14}$, $\boldsymbol{e7}$ to $\boldsymbol{e15}$, and $\boldsymbol{e8}$ to $\boldsymbol{e16}$. In the case study, this implies that the analysis performed for the first 8 states can be extended to the remaining 8 states, with the only difference being an inversion of signs in the motion vector (**MV**) of each variable.

4.4.2 Establishing a relationship between unlinked variables of the intermediate links

In column 2 of Table 1, the variable ω_{4y} represents the movement of the planetary gear (only one planetary gear is analyzed, as the other exhibits the same movement, making it a redundant link). This link follows a **GQS** of $\{+,0,-,0,+\}$. Within this interval, the variable is positive whenever the inequality $\omega_{3x} > \omega_{5x}$ is satisfied. $\omega_{4y} = 0$ when $\omega_{3x} = \omega_{2x} = \omega_{5x}$, whether these velocities are positive or negative. It describes a negative rotation when $\omega_{3x} < \omega_{5x}$. Additionally, it can be observed that ω_{2x} follows a **GQS** of $\{+,0,-,0,+\}$.

It is important to note that, during the analysis of states when constructing the MV, it is not possible to establish a direct relationship between the variables $\omega_{3x}, \omega_{2x}, \omega_{5x}$ and ω_{4y} using disambiguation operators. However, at this stage of the analysis, when considering the inverse envisioning, the role of disambiguations and the general matrix becomes evident in establishing relationships between all the variables.

Fig. 9 illustrates a vector diagram which corresponds to the 17 qualitative states that comprises an entire cycle of movement (horizontal axe) and the angular velocity of the links of the mechanism, i.e., the general behavior of the qualitive variables of the mechanism, based on the **GMM** (except for the variable ω_{4y}). At the end of the cycle, the qualitative movement of the mechanism returns to the initial state ($e1$). This representation is similar to the one used by Han & Lee (2002), with the key difference being that they represented linear displacement along three axes, which was generated by a software.

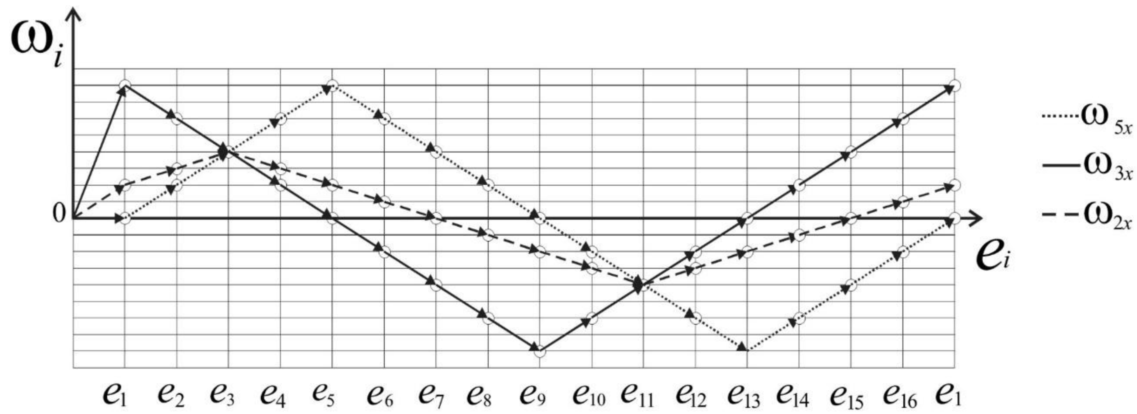

Fig. 9. Vector diagram corresponding to the envisionment of the differential gear train.

4.5 Generation of the conceptual design

4.5.1 Ratchet role: restriction of movements

As stated previously in Section 3.4, the ratchet is a critical mechanism in this case study due to its constraint function within the **GMM**. With the constraint function of the ratchet, we can control rotational movement in specific directions. A ratchet can isolate **QS** of a component in the mechanism, either by allowing rotation in only one direction (positive or negative relative to the QS), or by enabling relative movement, such as rotations at a speed equal to or greater than that of a given component. This capability facilitates the creation of new conceptual mechanisms by altering the continuity of movement in a given system.

The next consideration is determining where to place the ratchet within the mechanism. Due to its mechanical nature, it must be positioned in a physically permissible manner—either between two components with concentric axes or between an axis and its frame (conditions described in Equation **Error! Reference source not found.** and Equation **Error! Reference source not found.**). It cannot be placed between components that do not share an axis, such as between links 3 and 4, nor between components that are not in physical contact, like between links 3 (input) and 5 (output). In this case study, the required output rotation is always positive. That is, when link 3 rotates positively, link 5 will also rotate positively. Even when link 3 rotates negatively, link 5 will continue to rotate positively. This behavior is represented by the Equation (4):

$$\omega_{5x} \geq |\omega_{3x}| \quad \textbf{(4)}$$

The **QS** of the input link for the cases where $\omega_{3x} \geq 0$, and $\omega_{3x} \leq 0$ are **QS** $\{+,0,-\}$ and **QS** $\{-,0,+\}$, respectively. Meanwhile, the output link, which satisfies $\omega_{5x} \geq 0$, must have a **QS** $\{0,+\}$.

Since link 5 is not directly attached to the frame but rather to link 2, this implies that when $\omega_{5x} \geq 0$, it follows that $\omega_{2x} \geq 0$ as well. So, the angular rate ω_{5x} is transferred to link 2. Then, the ratchet will be placed between link 2 and frame 1. This eliminates the qualitative states from $\boldsymbol{e8}$ to $\boldsymbol{e14}$. It is important to note that if the requirement were $\omega_{5x} \leq 0$—that is, unidirectional but negative—, consequently $\omega_{2x} \leq 0$, the states from $e1$ to $\boldsymbol{e6}$ and $\boldsymbol{e16}$ would be removed by using a ratchet to restrict this direction of rotation. However, in qualitative states $\boldsymbol{e15}$ and $\boldsymbol{e16}$ $\omega_{2x} \geq 0$ but $\omega_{5x} < 0$. To eliminate these states, a ratchet can be placed between links 2 and 5, ensuring that $\omega_{5x} \geq \omega_{2x}$. By combining the actions of both ratchets (Equations (1) and (2)), can be established that:

$$\omega_{5x} \geq \omega_{2x} \geq 0 \quad \textbf{(5)}$$

The second ratchet eliminates states $\boldsymbol{e15}$ and $\boldsymbol{e16}$, as well as the qualitative states $\boldsymbol{e13}$, $\boldsymbol{e14}$, $\boldsymbol{e1}$, and $\boldsymbol{e2}$, leaving only states $\boldsymbol{e3}$ through $\boldsymbol{e7}$. The system is then reduced to states $\boldsymbol{e3}$, $\boldsymbol{e4}$, $\boldsymbol{e5}$, $\boldsymbol{e6}$ and $\boldsymbol{e7}$, in which the following conditions are met:

$$\omega_{3x} \leq \omega_{2x} \quad \textbf{(6)}$$

$$\omega_{4y} \leq 0 \quad \textbf{(7)}$$

It is important to note that placing the ratchets in these positions allows the gears to reverse from state $\boldsymbol{e7}$ to $\boldsymbol{e6}$ and then to $\boldsymbol{e5}$, as the ratchet was not positioned to restrict these transitions. As a result, the mechanism can continuously oscillate the input gear between positive and negative rotations without violating the conditions set by the inequalities of the qualitative states. The conceptual design of the mechanism described is shown in Fig. 10**.** The two isolated movement cases for link 3 are demonstrated, resulting in unidirectional rotation in link 5, as shown in Equation (4). It can be observed that the mechanism features two ratchets to limit movement—one between links 1 and 2, and another between links 2 and 3 (see equation (6)), ensuring the predicted motion is achieved. Gear 3 includes a pulley that facilitates the oscillating movement, while gear 5 incorporates a pulley that generates the unidirectional rotation. As suggested in Equation (7), link 4 always has an angular speed greater than zero.

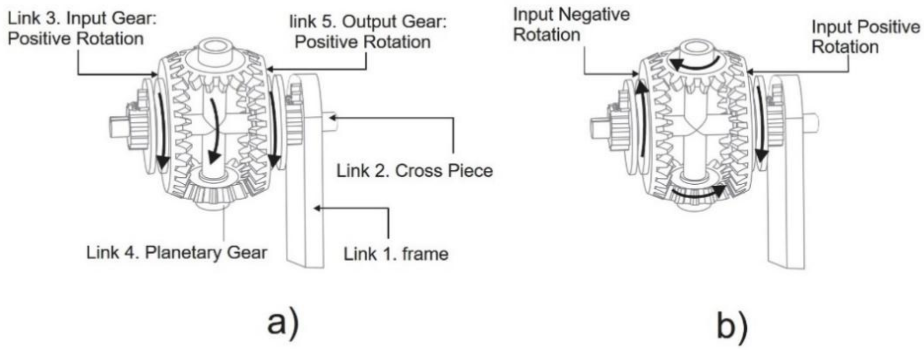

Fig. 10. Mechanism that converts oscillatory rotation into unidirectional rotation. In a), the system is shown with a positive rotation input, while in b), it is shown with a negative rotation input.

4.6. New envisionment

The new **GMM** that characterizes the mechanism is derived from the qualitative states $\boldsymbol{e3}$, $\boldsymbol{e4}$, $\boldsymbol{e5}$, $\boldsymbol{e6}$, and $\boldsymbol{e7}$ of the original **GMM**. To illustrate the mechanism's behavior graphically, a vector diagram can be constructed connecting these qualitative states, as shown in Fig. 11.

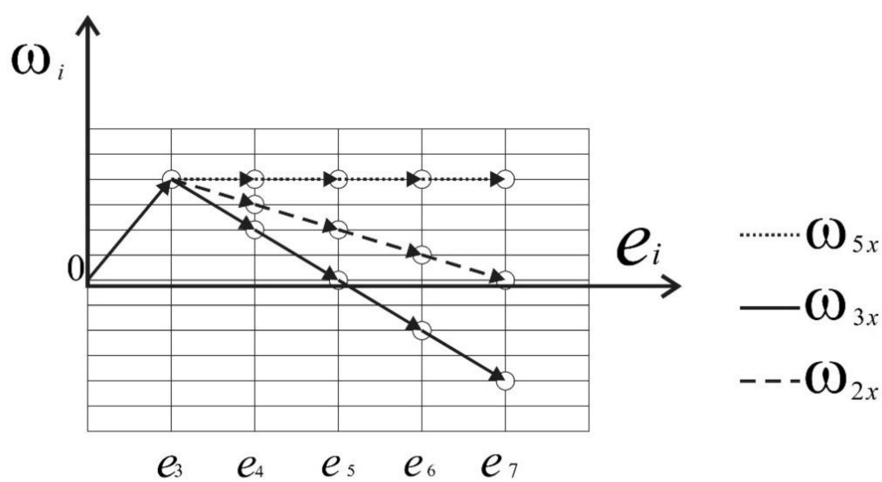

Fig. 11. Vector diagram corresponding to the envisionment of the qualitative states $\boldsymbol{e3}$, $\boldsymbol{e4}$, $\boldsymbol{e5}$, $\boldsymbol{e6}$ and $\boldsymbol{e7}$.

Since the mechanism is assumed to have continuous motion, the next qualitative state that would make the system return to the initial state is the state $\boldsymbol{e3}$. In Fig. 112 it can be observed that, connecting directly state $\boldsymbol{e7}$ and $\boldsymbol{e3}$ the mechanism describes the same behavior, but in reverse direction. Starting from $\boldsymbol{e7}$, the mechanism exhibits cyclic behavior, naturally transitioning back through the states $\boldsymbol{e6}$, $\boldsymbol{e5}$, and $\boldsymbol{e4}$. This graph shows the qualitative behavior of the mechanism, so the absolute values of $\omega_{3x}, \omega_{2x}, \omega_{5x}$ do not have a specific numerical value. This could influence in comparison against a numerical graphic of this mechanism since qualitative approach just offers an idea of its general kinematic behavior. It is important to note that, although this work focuses on a non-computational approach, each qualitative state and its associated set (isolated by a ratchet) function as a library or building block, as proposed by Kota & Chiou (1992). They can be used to construct a mechanism by integrating isolated movements, represented by individual qualitative states or their combinations.

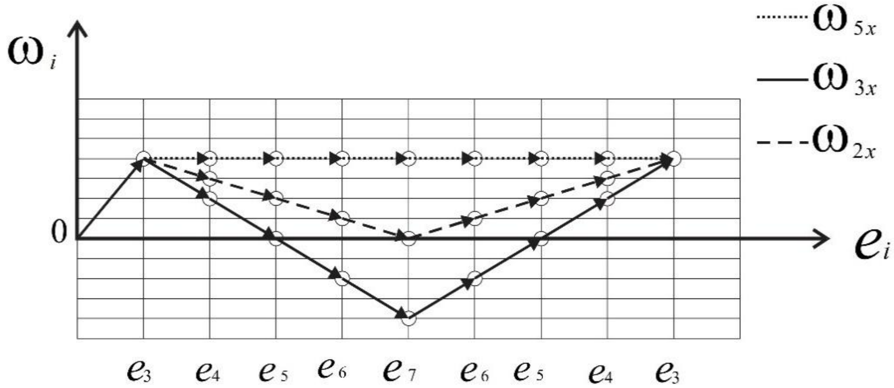

Fig. 12. Vector diagram corresponding to the envisionment of a cycle of oscillatory rotation.

4.7. Simulation in SolidWorks

The behavior of the mechanism described by the vector diagram of Fig. 12 is compared with a graph of a motion simulation obtained from the Computer-Aided Design (CAD) software SolidWorks. This comparison is shown in Fig. 13. The simulation describes the behavior of the mechanism when the input motion link is subjected to an oscillatory rotation. So, the behavior described by the variables ω_{5x} and ω_{2x} is a function of the rotation of link 3 (ω_{3x}). Note that Fig. 13 represents a range of movement of an oscillatory rotation cycle for link 3 with uniform amplitude. Being a simulation of the qualitative behavior of the mechanism, the absolute values of the angular velocities ω_{2x}, ω_{3x} and ω_{5x} are omitted in the CAD software. So, the diagram only describes the behavior of mechanism without specific numerical values of angular velocity. This is because we want to effectively compare it with the qualitative behavior shown in Fig. 12, and also because the angular velocity values in qualitative states generated in the vector diagram are relative and do not represent a specific numerical value. Thus, in the qualitative state $\boldsymbol{e3}$, which marks the starting point of the motion, the qualitative angular velocities ω_{2x}, ω_{3x}, and ω_{5x} align perfectly with the simulation, and the relationships described by the disambiguations in the other qualitative states are preserved, though with some differences. For instance, the motion initiation between states $\boldsymbol{e3}$ and $\boldsymbol{e4}$ for ω_{2x}, ω_{3x}, and ω_{5x} in the vector diagram is abrupt, whereas in the SolidWorks simulation, it is smoother. To obtain a vector diagram that more closely mirrors the behavior observed in a computer simulation, incorporating a greater number of quantitative subspaces in future work may be beneficial. Additionally, it is noted that for angular velocity ω_{5x}, the graph from the SolidWorks simulation displays an upward trend until it reaches its maximum value at $\boldsymbol{e2}$. This discrepancy with the vector diagram can be attributed to the fact that the qualitative analysis, unlike the SolidWorks simulation, does not account for the inertial effects of the gear train, which contribute to this behavior. This factor should be addressed in future research.

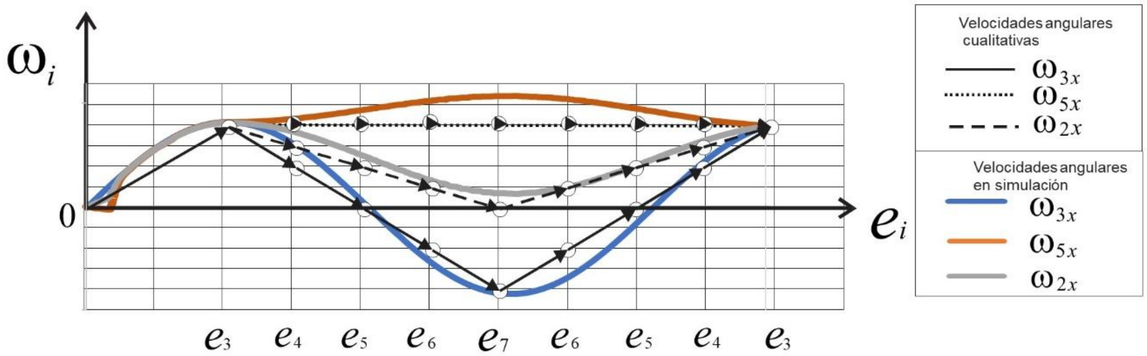

Fig. 13. Vector diagram vs Simulation obtained from SolidWorks.

5 Conclusions

The methodology developed in this work enables the conceptual design of a mechanism based on a qualitative reasoning approach, using a non-computational method. A qualitative description of the general behavior of a certain mechanism is generated based on its movement, which is called envisionment. This is discretized in qualitative states represented by a motion vector (**MV**), delimited by other qualitative states and differentiated by using signs and operators, which allow reducing ambiguities between qualitative states.

By linking the qualitative vectors in a logical sequence of motion, a general movement matrix (**GMM**) is created, which graphically represents the visible envisionment in a vector diagram. This matrix is used to analyze the relative motion between the components of the mechanism. By incorporating ratchets as restriction functions, specific movements can be isolated to form a new conceptual design.

This methodology is applied to a case study that focuses solely on the qualitative variables of rotation, specifically angular velocity. It was observed that the movement requirements for a converter mechanism, transitioning from oscillatory rotation to unidirectional rotation, fall within the range of movements that a differential gear train can produce. In other words, the input gear 3 undergoes oscillatory rotation, with its angular velocity ranging from $\omega_{3x} = \{-\infty, \infty\}$, while the angular velocity of the output link 5 is constrained to $\omega_{5x} = [0, \infty\}$. The envisionment of the differential gear train is then generated through the construction of its **GMM**. The **GMM** serves as a tool to determine the optimal placement of the ratchets, which isolate the vectors that define the required movements. These isolated vectors can then be connected to form the envisionment of the new mechanism, along with a corresponding vector diagram that represents it. The conceptual mechanism was produced in a CAD software and the motion of its main links was simulated. It was observed that the simulated motion and the qualitative motions vector diagrams developed agreed well.

While the methodology outlined in this study has been applied to a single mechanism, it possesses the potential for extension to other gear trains, such as epicycloidal and parallel configurations, as well as to mechanisms like the crank slider, four-bar linkage, and cam follower. Given that qualitative descriptions of the kinematics of mechanisms are inherently qualitative, this opens avenues for future research employing these principles in the training of advanced artificial intelligence systems for the conceptual synthesis of mechanisms. The selection of characteristics for the mechanism to be synthesized, along with the decision of which functions to constrain or isolate from existing mechanisms, remains at the discretion of the designer and their specific objectives.

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