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Improving the long-term cutting plan on a furniture company: A Case Study

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Abstract. In this paper we tackle a two-dimensional cutting problem to optimize the use of raw material in a furniture company. Since the material used to produce pieces of furniture comes from a natural source, the plywood sheets may present defects that affect the total plywood that can be used in a single sheet. The heuristic presented in this research deals with these defects and present the best way to handle them. It also considers the use of the plywood sheets for the long term planning of the company, since usually purchases of raw material are done only at certain periods of time, and must last for several weeks. Experimental results show how an intelligent cutting plan and selection of the plywood sheets reduce considerable the amount of raw material needed compared with the current operation of the company, and guarantees that the purchased sheets last during the planning period, regardless of the available area to cut pieces on each plywood.
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1 Introduction

The two-dimensional bin packing problem is the problem of packing a given set of rectangles into a larger one. This problem has many applications in real life industries, like glass cutting to manufacture windows; steel cutting to create smaller part, or wood cutting to create furniture. In all these industries, the cost of the raw material is high, thus optimizing the cutting to extract smaller parts is a crucial part in the reduction of costs. In many cases, when the bigger rectangle comes from a pre-manufactured material, we can guarantee homogeneous stock sheet where to extract the small pieces. However, when the raw material comes from a natural source, like wood, leather or fabrics, certain defects may appear in the stock sheet that prevent certain parts to be used.

In this paper, we concentrate on the problem of cutting wooden pieces from a rectangular piece of plywood in the furniture manufacturing industry. Being plywood a material manufactured with natural wood, it is common to find areas within the sheet of plywood with defects that cannot be present in the final product. These defects are randomly present in the stock sheet and their position is not known until the very moment the pieces are going to be cut from it. This fact adds an extra difficulty to the well-known problem of minimizing the number of stock sheet required to cut all the pieces, since every time we are cutting pieces from a new plywood sheet a new defect arises and we do not know nor the position of the defect, nor its size, thus we can assume that the stock sheets are all different.

When dealing with bin packing problems, even when they present defects, it is assumed that the number of stock sheets is infinite, or at least large enough to place all the pieces. However, when the defects arise randomly the decisions on how to pack a certain set of pieces may impact on the number of stock sheets needed from one packing pattern to the next, even if the set of pieces is the same. Companies, usually buy their stock to last for several weeks, and the cutting of pieces is made constantly, thus we need to ensure that the available stock will be sufficient until the next shipment arrives. It is also important to consider that inventory costs are high, so buying sufficient stock sheet to satisfy even the worst case scenario may not be feasible due to the high cost of holding inventory.

In this work, we also take this extra constraint in consideration, since the company where the problem arises has a weekly demand of certain sets of furniture, but since the stock sheet has a delivery leap time of 3 weeks, purchases of raw material should be enough to last these 3 weeks. In addition, due to the fact that defects arise naturally, we need flexible packing patterns that can satisfy the demand with the purchased material until the next shipment arrives.

This paper presents an efficient heuristic that minimizes the number of plywood sheet to produce furniture, taking into account the defects the stock sheet present, and checking the solutions obtained during a long horizon plan. Also, since this is a problem that arises from a real company, we are interested in giving the company the possibility of reusing the leftovers after the cutting process. Thus, our heuristic will try to place pieces so that the leftovers are as compact as possible, creating new reusable areas for future use. We will prove, through our experiments, that this heuristic, in combination with a simple local search over the selection of the plywood, can reduce the number of stock sheet used, and in some cases reaches the lower bound. On the other hand, we will show that the leftovers are such that can be reused in future cutting operations.

2 Preliminary Work

Two-dimensional bin packing problems (2DBPP) are problems that have been extensively studied over the years. According to [1] cutting and packing problems can be divided in two major groups: input minimization problems and output maximization problems. Input minimization problems are the ones where given a set of small items (pieces) we seek to minimize the number of large objects (bins) necessary to cut all the pieces. In output maximization problems, the objective is to select a subset of pieces to place in a given bin, so that the value of the selected pieces is maximized. If the value function of the pieces is proportional to their area, both problems are symmetric, however, in many applications this is not the case.

For this work, we are dealing with an input minimization problem, where we try to minimize the number of plywood sheet required to cut the necessary pieces to build some furniture. Regarding this problem, the literature has concentrated in finding the optimal way to place the pieces within the bin [2]. Most of the literature consider the case, where pieces have a given orientation and cannot be rotated. Although this is not our case, we must mention that all algorithms we show in this revision work under this assumption; otherwise, we will mention it.

Berkey and Wang [3], propose four heuristics to pack pieces: a *finite next-fit* (FNF), *finite first-fit* (FFF), *finite best-strip* (FBS) and *finite bottom-left* (FBL). Heuristics FNF, FFF and FBL pack directly into finite bins; whereas FBS is a two-step heuristic where first they pack in a strip of infinite height and then the strip is divided into blocks, which are then packed into finite bins. Essentially, the difference between *next-fit* and *first-fit*, is that the next-fit approach tries to fit the object in the current bin, if unsuccessful, it assigns a new bin for the object to be placed; and the first-fit places the piece in the very first bin that can accommodate the object. The *bottom-left* procedure searches, among all the available bins, for the lowest and left-most position where the piece can be placed. If there is no bin, where the piece can be placed, a new bin is made available. An exhaustive survey of classical placement procedures for bin packing problems can be found in [4]. None of these constructive heuristic takes into account defects on the bins.

Beasely [5] developed a binary linear program for the two dimensional bin packing problem with defects on the bins. However, the basis of the formulation lies in discretizing the bin into possible placing points. This technique clearly limits the precision of the placing, since there is a tradeoff between speed in the solution and precision on the placing. The more points we add to the grid the better the precision in the placing scheme, but the slower to obtain a solution. Martello and Vigo [6] proposed an exact method that adopts a two level branching scheme, the outer branch decision tree and the inner one, where the items are loosely assigned to the bins in the outer level and the inner branch decision tree enumerates possible sequences to finalize the positions of the items in the corresponding bin. However, in this case they do not consider defects on the bins.

Although these heuristic try to minimize the total number of bins, they do not pay attention to the arrangement of the leftovers. In our problem, this is an important feature, since we are interested in future use of these leftovers as a secondary objective. That is, even if we are not using leftovers in the present work, we are seeking for solutions where the leftovers can be used in the future, thus we need to pack the pieces so the waste is as compact as possible. Wang and Chen [7] use the concept of *residual space* defined as the largest rectangular area that can be obtained in a free area, after placing a piece [8]. They create a heuristic where they try to maximize the residual space on each bin, while the number of bins is minimized. The most critical point step in this heuristic is to determine the residual spaces (RS) defined once a piece is placed. This can easily be explained in the following example. In *Figure 1*(a), we can see that, after placing one piece in the bin, if we continue the edges of the piece until they reach the end of the bin, three new rectangles are created: R1, R2 and R3. However, none of these are the largest rectangles that can be obtained out of the free area, as we can see in *Figure 1*(b), where the two residual spaces are marked in dotted and discontinuous lines. These residual spaces where obtained by merging R1 and R2 in one rectangle, and also R2 and R3.

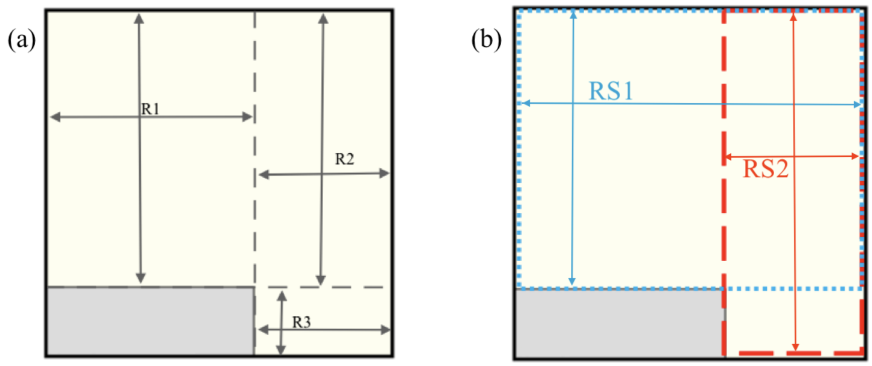

Figure 1: Definition of a Residual Space

The *Residual Space Maximized Packing* (RSMP) heuristic by Wang and Chen [7] uses this concept to select the best packing position for the next piece. This heuristic will be explained in detail along with the modifications we introduce to our problem in Section 4.

In the next section, we explain in detail the problem we are solving, and introduce the notation we will use all along this paper.

3 The Problem

The problem we are presenting in this paper is motivated by the lumber industry, where sheets of plywood must be cut in smaller pieces to complete the demand of certain pieces to assemble furniture. The objective is to use the minimum number of stock sheets to place all pieces needed to satisfy the demand. We also seek for placements where the largest rectangle that can be found in a bin after it is completely packed is as big as possible.

We denote by B the set of plywood sheets, and $B_i \in$B the i-th sheet where to place pieces. All plywood sheets are of the same height (H) and width (W), and the pieces $k \in$P to be cut are all rectangles with width (w_k) and height (h_k). We consider that all the pieces are different, and in case we have similar pieces, we list all copies of them. For this problem, we consider that the plywood sheets may present some defects in the form of an oval. Although the defects have the form of an oval, we consider its enclosure rectangle to represent it. This modification is not relevant in terms of the quality of the solution, since all pieces to be placed are rectangles, and if the cutting plan needs to avoid the defect, the space surrounding it will be a rectangle.

The plywood company, the one that supplies the raw material, consider different qualities of plywood in terms of the number of defects that may present. For this work we only consider high quality plywood, which means that at most one defect may be present in the sheet.

The second element in this problem are the pieces to be cut. All of them are rectangles, however, we must distinguish two types of pieces, the ones that cannot absorb the plywood defect (type 1) and those than it does not matter if the defect is present in the piece (type 2). The second type usually correspond to pieces that are hidden in the final assembled product. Thus, once we know the type of piece we are placing, the list of available RS is modified accordingly, as we can see in *Figure 2*, where we show how the set of possible RS changes depending whether the piece to be place can or cannot absorb a defect. In this example, the defect is represented as the dark blue rectangle in the middle of the bin. Note that, when a piece of type 1 is placed next, we need to consider five RS, whereas if the piece may absorb the defect only two RS need to be considered. This add new difficulties to the problem, since we need to consider different RS depending of the type of piece we are placing.

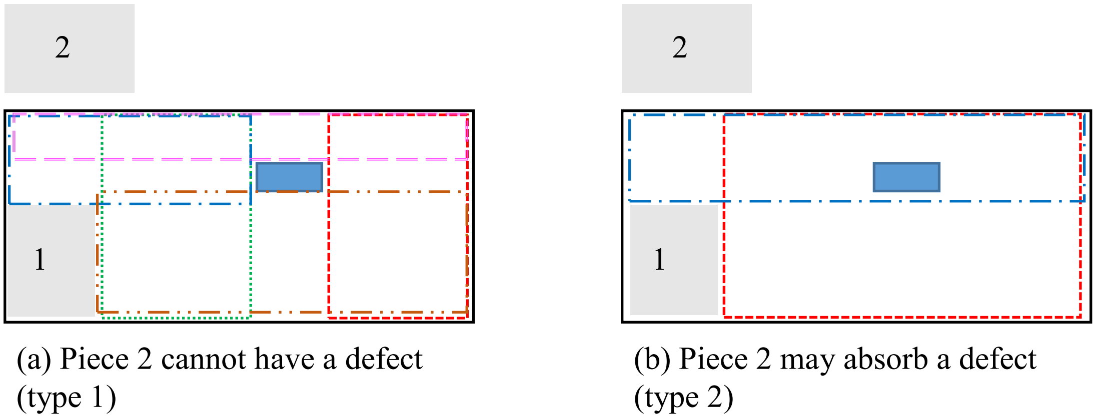

Figure 2: Residual Spaces when the next piece is from type 1 or type 2

In the next section, we explain in detail the heuristic to place the pieces taking into account all these particularities of the problem.

4 Residual Space Maximized Packing

In this section, we explain in detail the Residual Space Maximized Packing (RSMP) heuristic proposed by Wang and Chen [7]. Bigger pieces, are usually the most difficult to pack, since they can leave big unused spaces, producing waste. Thus, the authors propose a heuristic where the criteria to place pieces is by maximizing the residual space on each iteration.

The concept of "big piece" may be ambiguous in the context of rectangles, as a piece can be big in area, in perimeter, or in just one of the dimensions. Thus, the authors propose a preprocessing of the pieces rotating them so that the height is always less than or equal to their width ($h_k < w_k$, for all piece k∈ P), and then use three ordered sequences: order the pieces by height (breaking ties by choosing the widest), order the pieces by width (breaking ties by choosing the tallest); and order the pieces by area (breaking ties arbitrarily). For each piece in this list, it will have 8 possible placements inside a residual space, as can be seen in
Figure *3*. The RSMP heuristic selects the configuration that produces the biggest residual space in terms of area. Note that in this example, the resulting configurations with each rotation produce the same residual spaces; however, we need to take into account also the other residual spaces that the selected configuration may affect. This will be seen clearer once we explain this heuristic in more detail.

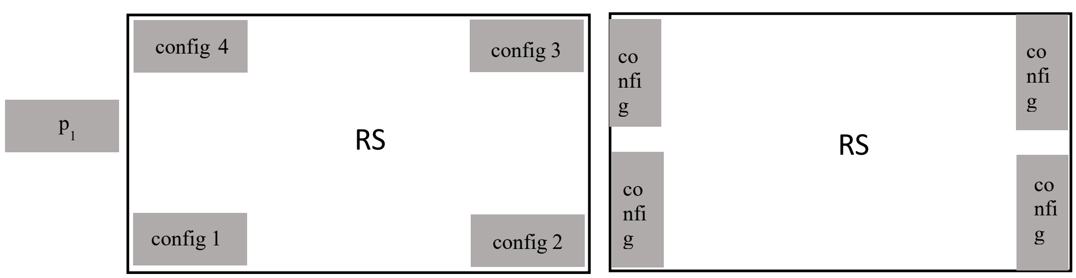

Figure 3: For a given piece, there are eight different configurations

The RSMP heuristic starts with a list H_j (j = 1, 2, 3) of pieces ordered by width, height and area, as explained before. Let RS_i the list of residual spaces available at sheet B_i. Let p_k be the k-th piece from list H_j. For all configurations, we try to place p_k into all the elements from RS_i. We select the placement that produces the biggest residual space in terms of area. Once piece p_k is placed in the bin, we need to update the list RS_i. If piece p_k does not fit in any of the residual spaces of the current bin B_i, we move to the next sheet where there is a residual space that fits. If it does not fit in any residual spaces, we need to use a new bin. The heuristic finishes when all pieces are placed for all three lists H_j, and selects the one that uses less number of bins.

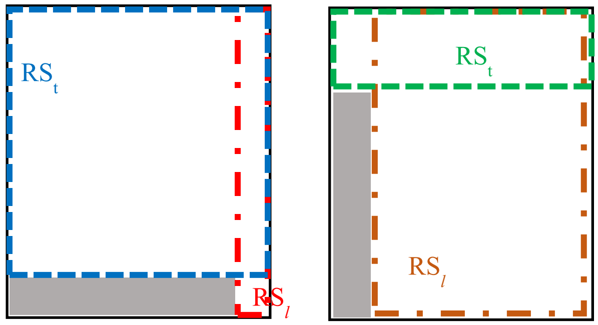

Figure 4: Residual spaces generated when piece p_1 is placed in config1 or config5

Let us illustrate this procedure with a small example. Let us consider a bin B with width $W = 210$ and height $H = 250$ units. Let us consider two pieces p_1 and p_2 with $w_1 = 180$ and $h_1 = 30$; and $w_2 = 90$ and $h_2 = 30$. Each residual space will be denoted with a pair of coordinates $[(x_1, y_1), (x_2, y_2)]$ that correspond to the bottom-left and top-right corners of the rectangle that defines the residual space. Since when placing piece p_1 only one residual space will be affected, only two configurations must be checked, config1 ($w_1 > h_1$) and config5 ($h_1 > w_1$), each of them creating two residual spaces. This can be seen in *Figure 4*, where piece p_1 is placed in the bottom left corner of the bin with both rotations. With these placements, the list of residual spaces has four elements, RS_1 = {[(0, 30), (210, 250)], [(180, 0), (210, 250)], [(0,180), (30, 250)], [(30, 0), (210, 250)]}, where the largest one is the one with corners at [(0, 30), (210, 250)] (the blue RS in *Figure 4*[1]). Thus the configuration that produces this residual space is considered, and piece p_1 is placed.

To place piece p_2, we select all the residual spaces generated after placing piece p_1 in the chosen configuration, let us call them rs_1= [(0, 30), (210, 250)] and rs_2 = [(180, 0), (210, 250)]. We need to place piece p_2 in all eight configurations in these two residual spaces, and check all the residual spaces created. When placing the piece in the first residual space rs_1, we obtain the residual spaces as shown in Figure 5 and *Figure 6*.

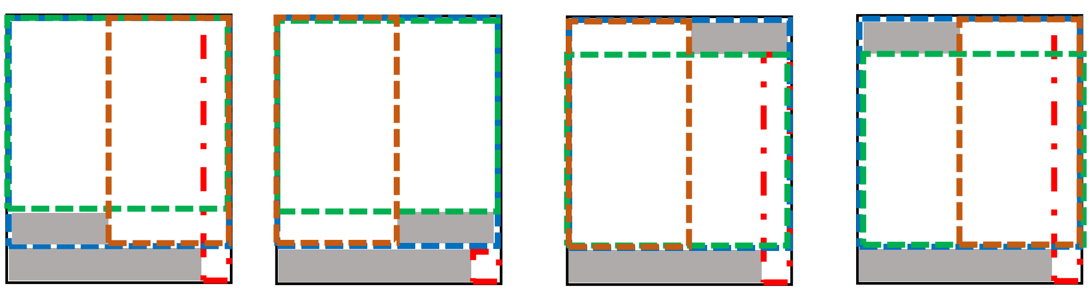

Figure 5: Residual spaces generated when piece p_2 is placed in rs_1 with original orientation ($h_2 < w_2$).

[1] The authors refer the reader to the online version of this paper for a coloured version of the illustrations.

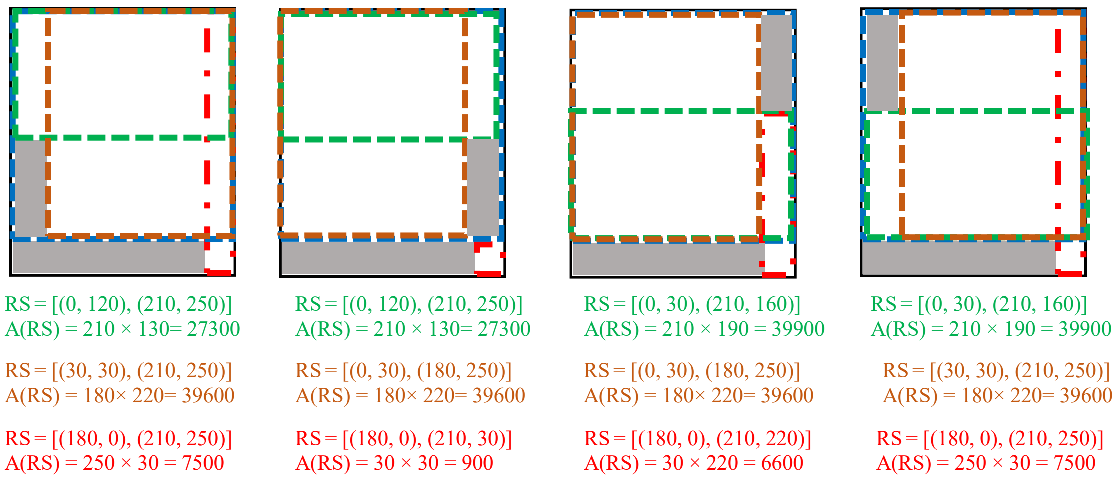

Figure 6: Residual spaces generated when piece p_2 is placed in rs_1 with piece rotated ($h_2 > w_2$).

As we can see the configuration that produces the largest residual spaces is to place the piece in the rotated position ($h_2 > w_2$) in the top left corner of rs_1. Note that although there is a tie between the two biggest residual spaces, it is the third RS in terms of area the one that breaks the tie. However, piece p_2 still need to be checked in rs_2. Note that in this residual space, p_2 only fits in the rotated position, and only two configurations must be checked: bottom and top. In

Figure 7 we can see the residual spaces generated with these positions:

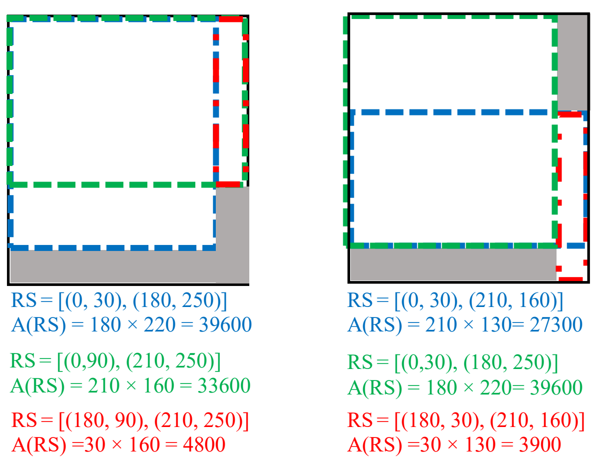

Figure 7: Residual spaces generated when piece p_2 is placed in rs_2 with piece rotated ($h_2 > w_2$).

Comparing the residual spaces generated by all ten possible positions, we conclude that the best placement for piece p_2 is to place the piece in the rotated position ($h_2 > w_2$) in the top left corner of rs_1. To place piece p_3, we would do the same analysis, placing the piece all three residual spaces that have been generated when piece p_2 was placed. If piece p_3 does not fit in any of the three residual spaces, then a new plywood sheet is needed to place that piece. Only when the dimensions of a given residual space are smaller than the smallest piece, we declare that residual space as waste.

4.1 Modifications to the RSMP heuristic to consider defects in the stock sheet

The original version of the RSMP considers that all the free space in the stock sheet is available to place pieces in it. In our problem, we need to deal with the defects that the stock sheet may present. In our problem, the stock sheet may have a defect, and the piece to be placed can or cannot absorb this defect when cutting it.

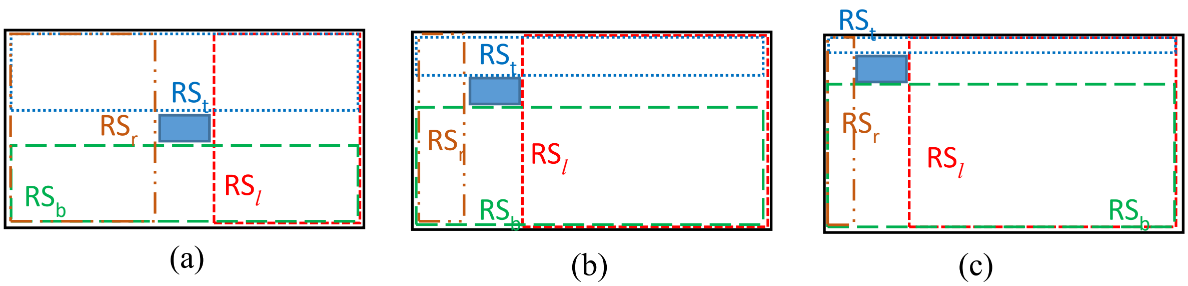

Figure 8: Different residual spaces created by a defect in a plywood sheet.

Defects will be treated as placed pieces, that can be anywhere in the plywood sheet. Although the defect is usually an oval shape, we consider its rectangle enclosure, and treat it like a piece already placed. The supplier may consider all plywood of the same quality, however, for practical purposes, we found that the position of the defect is important in determining the available space to place pieces. When a piece is placed in the middle of the stock sheet, four residual spaces are created: one on top of the piece RS_t, one at the left of the piece RS_l, one at the bottom of the piece RS_b and one at the right of the piece, as shown in *Figure 8: Different residual spaces created by a defect in a plywood sheet.* Note that the size of the largest RS varies with the position of the defect. This is important since the original RSMP is designed for identical bins, where the initial RS is the same. Thus, we order the bins by a measure for the quality of the bins, Q_i, that indicates how close to the corner of the bin is the defect, or defects presented. Note that, the closer to corner the defect is placed, the more difference we have between the larger RS and the smaller ones. We calculate Q_i as:

$$Q_i = \frac{A(RS_1)+A(RS_2)}{2A(B_i)} \qquad (1)$$

where $A(RS_1)$ and $A(RS_2)$ are the areas of the two largest residual spaces, and $A(B_i)$ is the area of bin B_i. As Q_i is closer to 1, it means that the area of the larger rectangles is closer to the area of the bin, thus, the available space to place pieces is close to the initial bin. We only consider the two largest rectangles, as the bigger they are, the smaller the other two are, and we are interested in calculating an average of the largest residual spaces. We will order bins by ascending Q_i, trying to use those bins with smaller RS's (worst quality first). This is an important modification to the original heuristic, since now we do not only have an order in the pieces but also in the bins where they will be placed. Changing the order of the bins may affect the final number of bins used, as will be explained in Section 5 where the experimental results are presented.

Recall that the RSMP heuristic uses three ordered lists to place pieces, and it then selects the one that gives the better results. Since we are introducing more variables to the problem, in order to keep the search space to a manageable size, we only order pieces by area. We also need to take into account whether the pieces can or cannot absorb the defects, thus rather than keeping two lists at all times, and update them, we found that it was more efficient to calculate the RS once we knew which type of piece will be placed.

Thus our implementation of the heuristic resembles an iterative process where we check the type of piece to be placed, the available residual spaces, and try to place the selected piece in any of the residual spaces. If the selected piece does not fit, then we use a new sheet of plywood. We describe the above process in more detail:

Step 1: Order pieces by decreasing area, breaking ties arbitrarily.
Step 2: Calculate Q_i for all $B_i \in$B and order the bins from smallest value of Q_i to the largest.
Step 3: For each piece $k \in P$:

- Create all RS for all $B_i \in$B.
- For all eight configurations, check the generated residual spaces for piece k in B_i (i = 1) and place this piece in the position that generates the largest residual space.
- If piece k does not fit in any residual space for B_i, then try to place this piece in i = i+1, and repeat the process until we can find a bin where piece k fits.

This initial heuristic tries to use first the worst bins in terms of quality, that is first the smallest values of Qi. This was done under the premise that the solution obtained under this scheme would be easily improved by changing the order of the bins. However, as we will see in the next section, results under this policy are not far from the lower bound.

4.2 Improvement phase

The constructive heuristic we presented for this problem, orders the plywood according to its quality, and uses the ones with lower quality first. Although results obtained with this heuristic were good and close to the optimal solution, we found some instances where the solution was far from the optimal. This may impact in future operations, since we need to remember that the company only has a limited number of plywood that it is was estimated to last for the three-week period until the next replenishment. Thus, we implement an improvement phase on those solutions where the lower bound was not achieved.

Note that a secondary objective is to find layouts where the final waste is as compact as possible and big enough that can be used in future operations. Thus, we introduce the idea of reusable waste (R_i) as the largest residual space that is left once all the pieces are placed in the stock sheet. Then, we implement an improvement over the pieces placed in sheets where the reusable waste is bigger than a certain threshold and try to accommodate these pieces in different sheets with better quality, ensuring that either the new waste will be greater than this value and therefore useful for future operations, or these subset of pieces can be placed in less sheets of plywood.

Our improvement phase can then be summarized as follows:

Step 1: From the solution obtained in the constructive heuristic, calculate the reusable waste (R_i) for each stock sheet. If $R_i >$ 15% of the plywood sheet, then extract the pieces from that bin.
Step 2: Order the unused bins in decreasing order of Q_i .
Step 3: Placed the extracted pieces in the bins as explained in 4.2.

With this improvement phase, we hope that for those cases where the lower bound was not achieved, we reduce the number of stock sheets needed, or, if that is not the case, the new placement will provide reusable waste with dimensions big enough to be used in the future.

Preliminary results showed that the improvement phase was successful in reducing the number of bins used. We present here some preliminary results obtained for an instance where the constructive solution used 11 bins. We can observe, in *Table 1: Quality of the plywoods used to pack pieces before and after the improvement phase* details for this instance. The first column on both tables represent the ID of the plywood once they have been ordered from worst to best in terms of our quality measure (Q_i), the second column represents the value of this quality measure and the third column the reusable waste on each bin. Notice that the lower the ID of the bin, the lower is its quality. The first thing we notice in this instance, apart from the reduction in the total number of bins used, is the quality of the bins. The initial heuristic needs to repack pieces from six bins, since its reusable waste is greater than 15%. It tries to repack these pieces in new sheets starting with the one with better quality (in our case bin 22). With this new repacking we are using less bins, but in contrast we are using those of higher quality.

Table 1: Quality of the plywoods used to pack pieces before and after the improvement phase

ID	*Qi*	*Ri*	ID	Q_i	R_i
1	0,53	3	1	0,53	3
2	0,54	19	4	0,58	13
3	0,57	18	17	0,76	15
4	0,58	13	18	0,76	24
5	0,59	18	19	0,77	15
6	0,60	35	20	0,95	8
7	0,64	56	21	0,95	8
8	0,66	56	22	0,96	10
9	0,66	56			
10	0,66	56			

11	0,67	56

Since the furniture company has to satisfy a steady weekly demand, and will not have more stock in the next three weeks, this use of high quality plywood may impact in the future and the company may run out of raw material to satisfy the three-week period demand. Thus, in order to make the solution viable for a three-week period, we implement a simple local search heuristic over the improved solution so that we guarantee the same number of bins but we seek to use those of low quality first, guaranteeing that the stock will last for the entire period until the next replenishment.

4.3 Local Search

We implement a simple descent local search heuristic over the quality of the bins, with initial solution the one obtained in the improvement phase. The idea behind this local search is to replace high quality bins with worse ones, without increasing the final number of bins. Since we do not want to increase the number of bins, we are repacking one bin at a time. Starting with the pieces in the bin with highest Qi, we try to pack the pieces in this bin in another one that is not already in the solution. If this is possible we replace the bin, and move to the next bin. This procedure is repeated until no improvement can be made. This procedure ensures the same number of bins, but attempts to use mid-quality bins that have not been selected in any of the previous phases.

Table 2: Quality of the plywood before and after the local search shows the impact of the local search, and how using the same number of bins we were able to use more bins of worse quality (bins 1 to 5), leaving the good ones for later in the period, and guaranteeing more space to place the same demand. Thus, ensuring that our stock of raw material will last for the entire planning period. Notice also that the reusable waste was reduced, on average. This can be explained since we are packing a set of pieces in the worst bin that fits, so since there is less effective space available to place pieces, if they fit in that bin, the waste is going to be reduced.

Table 2: Quality of the plywood before and after the local search

ID	Q_i	R_i	*ID*	Q_i	R_i
1	0,53	3	*1*	0,53	3
4	0,58	13	*2*	0,54	13
17	0,76	15	*3*	0,57	24
18	0,76	24	*4*	0,58	13
19	0,77	15	*5*	0,58	13
20	0,95	8	*17*	0,76	14
21	0,95	8	*18*	0,76	6
22	0,96	10	*19*	0,77	6
Average R_i		12		*Average R_i*	11,5

In the next section we present some results obtained with the heuristics we present here, and the impact each improvement has on the final solution.

5.- Experimental Results

In order to evaluate our heuristic, we use data from a furniture company. We consider a constant demand of 3 types of furniture, which need a total of 155 pieces. These pieces have different thickness, and can only be cut in plywood of the designated thickness. In our case we work with 4 types of plywood according to their thickness: 9, 12, 15 and 18 mm. To assess the goodness of our heuristic, we calculate a lower bound on the number of plywood sheets needed to place the pieces. This lower bound is calculated as if a perfect fit where possible, that is, assuming that every piece can be placed in any residual space whose area is greater than the area of the piece, regardless of the dimensions. Achieving this lower bound will mean that our solution is optimal. To calculate the lower bound, we compute the total area of the pieces $\sum_{k=1}^{m} A(k)$ and divided by the total available area, which is calculated as the total area of the bins, $\sum_{i=1}^{n} A(B_i)$, minus the area of the defects, $\sum_{d=1}^{D} A(d)$.

$$L = \left\lceil \frac{\sum_{k=1}^{m} A(k)}{\sum_{i=1}^{n} A(B_i) - \sum_{d=1}^{D} A(d)} \right\rceil \qquad (2)$$

We also need to include some data in terms of the quality of the bins, with respect to its Q_i. As mention before, the supplier measures the quality of their material by the number of defects and the probability that the defects may appear. However, we have shown that for our practical purposes, we need to take into account the position of the defect. For this initial tests we consider that the defects are evenly distributed around the plywood and on average $Q_i = 0.65$.

We first present the results of the constructive heuristic without the implementation of the improvement phase. Since we have four different types of plywood in terms of thickness, and each piece can only be cut from a certain type of plywood, we consider each thickness as an independent instance.

Table 3: Initial results obtained with the constructive heuristic

Plywood Thickness	Lower Bound	Number Of Bins	Minimum Reusable Waste (%)	Maximum Reusable Waste (%)
09 mm	3	3	15	17
12 mm	8	11	3	52
15 mm	9	9	3	28
18 mm	3	4	5	63

Table 3: Initial results obtained with the constructive heuristic shows the results obtained with the initial heuristic, when plywood sheets are ordered in ascending order of their quality. We notice that for two instances we achieve the lower bound (09 mm and 15 mm). These two instances also coincide with the ones with minimum values of the maximum reusable waste. From this initial solution, we apply the improvement phase only on instances 12 mm and 18 mm. Notice that, on both cases there are bins with a very tight packing, as the minimum reusable waste is 3% and 5% respectively. However, we also observe that the maximum reusable waste in both cases is high, thus, we repack the pieces allocated to those bins in order to reduce the final number of stock sheets.

Once the improvement phase is applied we observe, in *Table 4: Results after applying the improvement phase*, that the lower bound was achieved in all cases, and the size of the maximum reusable waste reduced.

Table 4: Results after applying the improvement phase

Plywood Thickness	Lower Bound	Number Of Bins	Minimum Reusable Waste (%)	Maximum Reusable Waste (%)
09 mm	3	3	15	17
12 mm	8	8	2	24
15 mm	9	9	3	28
18 mm	3	3	5	15

The improvement phase obtains the best solutions in terms of number of bins for each instance. However, in some cases, it uses high quality plywood. The biggest reduction in number of bins used occurred in the 12 mm instance, where an initial number of 11 sheets of plywood was reduced to 8.

To observe the effect of the local search, we need to look at the long term planning period. In our case, we consider a 12-week horizon with weekly demand and stock replenishment every third week (except for the first and last period that are of 2 and 1 weeks respectively). The data we used is displayed in *Table 5: Data for a 12 week period with replenishment every three week.* were we observe the different demand in terms of pieces and the available raw material (stock) on each week. The number of plywood available (stock) reduces according to the current operation, and increases on weeks 3, 6, 9 and 12, since the new stock arrives. For the long term period, we do not consider the different thickness of the plywood, and simply look at the overall operation.

Table 5: Data for a 12 week period with replenishment every three week.

Week	1	2	3	4	5	6	7	8	9	10	11	12
Pieces	165	84	74	143	106	49	69	165	168	222	74	49
Stock	57	31	63	52	31	94	85	73	112	86	53	41

One of the objectives was to evaluate the impact on the stock once the heuristic was applied, and corroborate that if we order less raw material, this will last over the planning period until the next replenishment. Since this research was motivated by a real company, and we are using their data, we compare our results with the number of stock sheets currently used.

Table 6 Comparative between the current operation and the proposed heuristic

Week	1	2	3	4	5	6	7	8	9	10	11	12
L.B.	24	13	10	19	15	9	11	24	25	31	10	9
C.O.	35	19	14	28	22	11	15	35	34	47	14	11
RSMP Mod	26	14	11	21	18	**9**	12	26	26	33	12	**9**
Improvement (%)	25.7	26.3	21.4	25	18.2	18.2	20	25.7	23.5	29.8	14.3	18.2

*Table 6 Comparative between the current operation and the proposed heuristic*shows the results obtained with the proposed heuristic (RSMP Mod) over a 12 week period, and compare these results with the current operation (C.O.). We also show the value of the lower bound (L.B.) calculated as in (2) for each week. We observe how the proposed heuristic obtains the optimal solution in two weeks, and in the worst case uses 3 bins more than the lower bound. The improvement obtained with respect to the current operation is noticeable and it translates in a savings of 68 sheets of plywood over the 12 weeks. The percentage of improvement is calculated in comparison with the current operation (C.O.). On average this improvement is of 20.6% and within a range between 14% and 29.8%.

Finally, we are testing our model with plywood of higher quality with respect to the value of Q_i. We now assume that the position of the defects is such that the average Q_i is 0.9. With better quality plywood we expect to obtain better solutions and reach the lower bound more often. We show the results in *Table 7 Comparative between the curren operation and the proposed heuristic with playwood of quality* $Q_i = 0.9$ where we can observe, as expected that the improvement has increased in comparison with the current operation, and now the range of improvements move between 18.2% and 31.6% with an average of 24.7%. We also achieve the lower bound in three occasions, and the total improvement in terms of sheets of plywood for the 12-week period is of 73. In comparison with the results presented in table *Table 6 Comparative between the current operation and the proposed heuristic*where the average quality of the plywood is of Q_i=0.65 the improvement is only of 2.3%, so low that it is not worth to pay more to ensure raw material of better quality.

Table 7 Comparative between the curren operation and the proposed heuristic with playwood of quality $Q_i = 0.9$

Week	1	2	3	4	5	6	7	8	9	10	11	12
L.B.	24	13	10	19	15	9	11	24	25	31	10	9
C.O.	35	19	14	28	22	11	15	35	34	47	14	11
RSMP Mod	26	**13**	**10**	20	18	**9**	12	26	26	33	**10**	**9**
Improvement (%)	25.7	31.6	28.6	28.6	18.2	18.2	20.0	25.7	23.5	29.8	28.6	18.2

5.1 Use of the Reusable Waste

One main reason to use the selected heuristic to approach the problem, was to ensure that the non-used space was as compact as possible. That is, ensuring that the largest residual space after placing all possible pieces in a bin, is large enough that could be reused in future operations. We refer to this residual space as reusable waste (R_i), as stated in Section 4.2 *Improvement phase* We have not included the utilization of these residual spaces in our heuristic, however, it is interesting to study whether it will be possible to do in future operations. According to the demand of the company, and the pieces that they are most likely to cut in the future, that a leftover can be reuse if its dimensions are larger than 20×60 cm.

In *Table 8* we present the results concerning to this measure. To show the results, we state, in the second row (*No. of* R_i), how many sheets, out of the total number of bins needed for the operation, have a reusable waste, and in the last row (% Area) we represent which percentage of area it represents with respect to the area of the bin. To calculate this percentage, we sum over all the areas of the R_i and divided over the area of all the bins. With these measures, we give the company an idea of how many R_i's are generated, and which is the mean area of these R_i's, so they can consider using them or not for future operations.

Table 8 Number of residual spaces that can be used, and average percentage of size

Week	1	2	3	4	5	6	7	8	9	10	11	12
Bins Needed	26	14	11	21	18	9	12	26	26	33	12	9
No. of R_i	3	2	3	6	7	2	6	7	8	9	5	3
% Area	23	14	49	15	37	31	30	13	18	23	30	14

We can see that there are, in total 61 reusable residual spaces, and its average size is 24% of the total bin. This is a good measure the company can take into account should they decide to reuse the leftovers. The fact that on average the size is over 20%, means that the residual spaces left to use are not that small and many different pieces may be allocated in them in future operations. We also observe that there is no correlation between the number of Ri's and the percentage of area. What it is interesting is that, on every week there is at least 2 bins where the company could reuse their leftovers for future operations, and that in many cases, when the number of reusable leftovers is greater than 7, the percentage of area is small enough implying that the packing is quite compact.

Although the use of leftovers is rare in practice, this analysis shows that with an efficient holding of inventory the company may save money in the purchase of raw material. However, to understand the impact of the use of these leftovers, we would need a new different study.

6.- Conclusions

In this paper we have presented a heuristic used to solve the cutting problem of a furniture company. This heuristic is a three-stage heuristic based on the residual space maximized problem. It first constructs a solution where all the pieces are placed given a initial order for the bins. Then, based on the fact that the bins may present defects, we try to reduce the number of bins by allocating a subset of the pieces in different bins. This improvement phase ensures that the number of bins used, is the minimum with the selected placing heuristic. However, since we have a limited amount of raw material that should last for a given planning horizon of 3 weeks, and a steady weekly demand, we want to guarantee that the available bins last for these 3 weeks. To do so, and starting with the improved solution, we perform a simple descend local search, where we interchange plywood sheets of good quality for others of not-so-good quality, and check if we can relocate the pieces without increasing the total number of sheets used.

With this heuristic we showed that not only we improved the current operation of the company by saving more than 65 plywood sheets over a 12-week period, but we also guarantee that for some weeks, the solution obtained was optimal.

Finally, we perform a last analysis based on the possibility of using the leftovers. This last check confirm that our heuristic packs the pieces in a way that the leftovers are as compact as possible, making possible for the company to reuse them, since in many cases are large enough to hold some of the most demanded pieces. Performing an analysis on how to use the leftovers is a complete different problem, and companies are usually reluctant to do so, since the operations costs may be higher compared with the estimated savings. They rather charge the customer for the used material, than estimate the savings of a more efficient way of handling leftovers.

However, many manufacturing companies are heading to operations with zero-waste, and being eco-friendly by reducing the use of raw material. Soon there will be certificates for eco-friendly companies and zero-waste operations. Thus, investigating the impact that the use of these leftovers may have in the overall production might be a way to start convincing them of the advantages of implementing ways of handling the inventory in a way that the leftovers are easier to handle than is now, so the savings in operation can be easily address.

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