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Analyzing valid bounds for a facility location bilevel problem with capacities

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Abstract. In this paper, valid bounds for a facility location bilevel problem with capacities are proposed. This problem arises from the situation when a company aims to locate some facilities such that the location and distribution costs are minimized. Nevertheless, the customers are free to choose the facility they prefer for satisfying their demand. Under this assumption, this problem can be modeled as a bilevel program, in which, the upper level is associated with the company's decision and the lower level corresponds to the allocation of the customers based on their preferences. The resulting lower level problem is NP-hard, which complicates the resolution of the bilevel problem due to the difficulty of obtaining –in general- feasible bilevel solutions. Hence, we explore other approaches for handling this issue. By considering traditional bounds for the lower level problem, we can propose valid bounds for the bilevel one. However, the impossibility of classifying them as upper or lower bounds is shown through computational experimentation.
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1. Introduction

Location science is an area that has always attracted the attention of the researchers due to their significant impact when dealing with real applications. Also, the complex nature of that kind of problems has motivated to the scientific community for studying them. Under the classical approach, customers are allocated -in a deterministic way- to their nearest located facility. However, in some situations that allocation criterion could be substituted by some others, such as, a specific utility function, customer's preferences or even, stochastic rules can be considered.

In this paper, we are interested in the criterion that allocates the customers to their most preferred facility considered in [1]. By following this allocation criterion, the facility location problems can be naturally modelled as bilevel programs. For instance, bilevel models for locating facilities without considering their capacity are studied in [2, 3, 4]. Also, in [5, 6] the constraint that forces to locate exactly p facilities isincluded to the bilevel problem.

In all the above-cited papers the lower level problem is an integer problem in which the optimal solution of its linear relaxation coincides with the optimal solution of the integer problem –due to the property of the closest assignment constraints, see [7]-. Hence, it could be solved via a commercial optimizer or exact procedures based on a reordering of the matrix of the customer's preferences, as in [8].

The remainder of this paper is as follows. Section 2 presents the background necessary to understand the importance of having the lower level's optimal response and an overview of some cases in which is not possible or computationally convenient. In Section 3 the mathematical formulation of the problem herein considered is presented. Section 4 introduces two schemes for obtaining valid bounds for the bilevel problem. The first scheme omits the lower level's objective function from the bilevel formulation. The second scheme takes into account the complexity of the lower level problem and relaxes the binary nature of its decision variables. Computational experimentation was conducted for measuring the quality of the bounds, and the results are shown in Section 5. Finally, the conclusions are stated in Section 6.

2. Background

As it was mentioned before, a bilevel programming problem is composed of two decision levels –an upper and a lower level- that interact with each other in a hierarchized manner. The existing relationship among both levels yields that the decision variables associated with the lower level are given by the optimal solution of another optimization problem parameterized on the upper-level decision variables. In other words, a bilevel programming problem is a mathematical program with another mathematical program in its constraints. Let $F, f: R^n \times R^m \rightarrow R$ be the upper and lower level objective functions, respectively. Defining$G: R^n \times R^m \rightarrow R^p$ the upper level constraints and $g: R^n \times R^m \rightarrow R^q$ the lower level constraints; a general mathematical formulation is as follows (see [9, 10]):

$$\min_{y}\{F(x,y): y \in R^m, G(x,y) \leq 0, x \in P(y)\} \quad (1)$$

where $P(y) = \{x \in R^n: x \in \operatorname{argmin}\{f(\hat{x},y): g(\hat{x},y) \leq 0\}\}$. Note that the constraint $x \in P(y)$ forces bilevel feasible solutions to have the particularity that a subset of its variables must be determined by the optimal solution of the lower level problem.

Nevertheless, due to the complex nature of some bilevel problems the optimal solution of the lower level cannot be always reached. For example, when it corresponds to a multi-objective problem, there is a Pareto front of solutions, not only a single optimal solution. See [11] for a detailed explanation about the issues that appear under this scheme. Another example is when a Nash equilibrium is considered at the lower level as in [12], in which, the lower level consists of several problems interrelated. The Nash equilibrium is assumed to be unique, but in general, that is not always possible. Moreover, when the lower level is an NP-hard problem –as the problem herein considered-its optimal resolution cannot be always guaranteed. For instance, in [13] a heuristic approach that reaches good lower level solutions were considered. In that problem, a minimum spanning tree must be obtained for having feasible bilevel solutions. Note that in that problem, the lower level is an NP-complete problem. Therefore, in a strict sense, the obtained solutions are bilevel infeasible in the mentioned cases. However, the problems under study need to be solved efficiently and acceptably.

Furthermore, some efforts have been made for studying the behavior of algorithms that are designed for solving bilevel problems. In other words, the impact of considering approximate solutions when solving the lower level problem instead of optimal ones has been analyzed. For example, in [14], a bilevel production-distribution problem is considered and the affectation on the upper level's solution when comparing an optimal procedure against an ant colony optimization algorithm is shown. Also, in [15] a similar analysis is made for a facility location bilevel problem without capacities, in which, the lower level was solved via an exact procedure and a heuristic one. In both cases, the findings are consistent that there is not a reliable manner to predict the impact on the upper-level objective function value when the lower level is not optimally solved.

Based on the above mentioned main issues, we propose valid bounds for the bilevel problem herein considered due to the impossibility of obtaining the lower level's optimal solution in the general case. Additionally, a discussion about some counterintuitive findings are presented and illustrated by computational experimentation.

3. Facility location bilevel problem with capacities

First, let us introduce the notation used in the model.

Decision variables:

y_i represent whether a facility is located in the potential location i (upper level).
x_{ij} denote if the demand of the customer j is supplied by the facility i (lower level).

Parameters:

$i = \{1, 2, \dots, n\}$, is the set of potential location for the facilities.
$j = \{1, 2, \dots, m\}$, is the set of customers that will be served by the facilities.
c_{ij} associated costs for allocating a customer j to the facility i.

f_i fixed cost for locating a facility at the $i-th$ potential location.
d_j demand associated to each customer j.
b_i production capacity to each facility i.
p_{ij} preference of the customer j towards the facility i, in which, if $p_{ij}=1$ indicates the less preferred facility and $p_{ij}=n$ corresponds to the most desirable facility.

The mathematical model is as follows:

$$\min_{y,x} \sum_{i=1}^{n}\sum_{j=1}^{m} c_{ij}x_{ij} + \sum_{i=1}^{n} f_i y_i \tag{2}$$

subject to:

$$y_i \in \{0,1\} \quad \forall i \tag{3}$$

$$x \in \operatorname{Arg\,max} \sum_{i=1}^{n}\sum_{j=1}^{m} p_{ij}x_{ij} \tag{4}$$

subject to:

$$\sum_{i=1}^{n} x_{ij} = 1 \quad \forall j \tag{5}$$

$$\sum_{j=1}^{m} d_j x_{ij} \le b_i y_i \quad \forall i \tag{6}$$

$$x_{ij} \in \{0,1\} \quad \forall i,j \tag{7}$$

where equation (2) denotes the upper level's objective function, in which the minimization of the locating and distributing costs is aimed. In equation (3) the binary nature of the upper level's decision variables is required. Constraint (4) shows the freedom that customers have concerning being allocated to the facility they preferred. That constraint is known as the lower level's objective function and indicates the maximization of the customer's preferences. The requirement that whole customer's demand must be met is guaranteed by equation (5). The capacity constraint associated with facilities is considered in equation (6), where the located facilities will supply the demand of all the possible customers without exceeding their production capacity. Equation (7) denotes the binary constraint associated with the lower level decision variables.

It is important to highlight that since (2) is being minimized for both decision variables, but y is decided in the upper level problem and x in the lower level one. This fact yield us to consider the optimistic version of the bilevel problem. In other words, if the lower level problem has multiple optimal solutions for a given upper level decision y; then, the resulting solution$x$ will be the one that results more convenient for the upper level's objective function. It is clear that the lower level's objective function will remain as the same. In this sense, it could be seen that there is a certain type of cooperation among both decision levels.

4. Obtaining the bounds

Due to the lack of commercial software capable of solving the bilevel problem described in the previous section, other resolution schemes are explored. One of the most common approaches is to reformulate the lower level problem via it corresponding KKT conditions. Also, a similar approach that uses the primal-dual relationships of the lower level problem is commonly used. Both approaches reduce the bilevel problem into a single-level one. Moreover, these approaches lead us to equivalent single-level problems. However, it is evident that these approaches cannot be applied straightforwardly to the problem herein considered due to the integrality constraints associated with the decision variables. On the other hand, a single-level reduction that obtains classical lower bound consists in omit the lower level's objective function and solve the remaining problem. Nevertheless, the quality of the resulting bound is poor in most of the cases.

Other resolution approach consists in developing an algorithm for handling the upper-level decision variables, while considering the lower level's resolution within the process. The selected algorithm depends on the problem's characteristics. For example, enumerative, heuristic or metaheuristic algorithms have been applied for solving bilevel problems.

We conduct two of the mentioned approaches, that is, reformulate the bilevel problem by considering a linear relaxation of the lower level and implement an enumerative algorithm for comparing the values given by the reformulation. In the latter algorithm, the lower level is optimally solved by an optimizer for each upper level's decision. By doing this, feasible bilevel solutions are obtained and the optimal solution is guaranteed.

4.1 Classic lower bounds of the bilevel problem

As it is mentioned above, a classical bound for bilevel problems consists in omitting the lower level's objective function and aggregate all its corresponding constraints into the upper-level problem. Since we are considering a minimization problem, the resulting bound is a lower one. By doing the latter, the upper level's decision maker will decide both decision variables, his and the lower level ones. Hence, he will select the best decisions based on his objective function without regarding the lower level.

Therefore, the single-level reformulation used for obtaining classical lower bounds of the problem (2)-(7) is as follows:

$$\min_{y,x} \sum_{i=1}^{n}\sum_{j=1}^{m} c_{ij}x_{ij} + \sum_{i=1}^{n} f_i y_i \tag{8}$$

subject to:

$$y_i \in \{0,1\} \qquad \forall i \tag{9}$$

$$\sum_{i=1}^{n} x_{ij} = 1 \qquad \forall j \tag{10}$$

$$\sum_{j=1}^{m} d_j x_{ij} \leq b_i y_i \qquad \forall i \tag{11}$$

$$x_{ij} \in \{0,1\} \qquad \forall i,j \tag{12}$$

This scheme is very useful to validate the bilevel structure of a problem. This is, if the optimal solution of the single-level problem (8)-(12) is the same than the optimal solution of the bilevel problem (2)-(7), then it implies that the most preferred facilities coincide with the closest ones; and the bilevel formulation lost it sense. Moreover, since the opinion of the lower level decision maker is not being taken into account, the upper-level objective function cannot be affected in any way. Hence, the optimal solution of the problem (8)-(12) will be a valid lower bound for the bilevel problem.

4.2 Proposed bounds based on the linear relaxation of the lower level

Consider the lower level problem defined by (4)-(7) and consider the linear relaxation of equation (7), that is, $x_{ij} \geq 0$ –this will be denoted as (7_{LR})-. Note that after this linear relaxation, the NP-hardness associated with the lower level is lost. Moreover, the primal-dual relationships of the resulting problem can be obtained. First, let $u_j (j = 1, \ldots, m)$and$v_i (i = 1, \ldots, n)$be the dual variables associated with the lower level. Then, its corresponding dual problem is as follows:

$$\min_{u,v} \sum_{j=1}^{m} u_j + \sum_{i=1}^{n} b_i y_i v_i \tag{13}$$

s.t.

$$u_j + d_j v_i \geq p_{ij} \qquad \forall i,j \tag{14}$$

$$v_i \geq 0 \qquad \forall i \tag{15}$$

Then, this reformulation consists in assure primal and dual feasibility of the lower level and include a criterion for guaranteeing optimality, such as, the slackness complementarity constraints or the equality of both objective functions. Therefore, the reformulated relaxed problem is given by (2), (3), (5)-(6), (7_{LR}), (14)-(15) and (4)=(13). Note that the latter equality is referred to the value associated with the primal and dual's objective function. Moreover, it can be appreciated that (13) is non-linear, but it can be easily linearized by including an auxiliary variable $z_i = y_i v_i, (i = 1, \dots, n)$. This is possible due to that y_i is a binary variable despite the fact that v_i is a continuous one. Hence, $z_i = v_i$ when $y_i = 1$; and $z_i = 0$ when $y_i = 0$.Taking advantage of these relationships, the proper constraints could be added into the model for linearizing it. A detailed description of this procedure but applied to another bilevel problem can be seen in [4].

Then, the resulting single-level mixed integer programming problem -in which M is a positive and sufficient large constant- is presented next:

$$\min_{y,x,u,v,z} \sum_{i=1}^{n} \sum_{j=1}^{m} c_{ij} x_{ij} + \sum_{i=1}^{n} f_i y_i \tag{16}$$

Subject to:

$$\sum_{i=1}^{n} x_{ij} = 1 \qquad \forall j \tag{17}$$

$$\sum_{j=1}^{m} d_j x_{ij} \leq b_i y_i \qquad \forall i \tag{18}$$

$$u_j + d_j v_i \geq p_{ij} \qquad \forall i,j \tag{19}$$

$$\sum_{i=1}^{n} \sum_{j=1}^{m} p_{ij} x_{ij} = \sum_{j=1}^{m} u_j + \sum_{i=1}^{n} b_i z_i \tag{20}$$

$$z_i \geq 0 \qquad \forall i \tag{21}$$

$$z_i \leq M y_i \qquad \forall i \tag{22}$$

$$z_i \leq v_i \qquad \forall i \tag{23}$$

$$z_i - M y_i \geq v_i - M \qquad \forall i \tag{24}$$

$$y_i \in \{0,1\} \qquad \forall i \tag{25}$$

$$x_{ij} \geq 0 \qquad \forall i,j \tag{26}$$

$$v_i \geq 0 \qquad \forall i \tag{27}$$

It is important to remark that this reformulation is not equivalent to the original bilevel problem. The main issue is that equation (7_{LR}) does not guarantees that the optimal solution be integer. Moreover, since the lower level is a maximization problem and it is being relaxed, the objective function value for this relaxation will be greater than the objective value for the integer problem. And, due to the fact that the optimal solution for the lower level parameterized in a fixed upper level variable will represent the approximation of the bilevel feasible region (inducible region). It is natural to suppose that the approximated inducible region is being overestimated. Then, the upper level decision will take place over that overestimated region and its minimum value will be greater than the optimal for the original problem. So, we could expect that the lower level's relaxation yield us to an upper bound. However, this is not always true (this is shown in next Section) since the absence of the lower level optimal solution will affect in a non-predictable manner the upper level objective function of the problem herein considered. On the other hand, this linear relaxation obtains good bounds, although they cannot be classified as upper or lower bounds.

5. Computational experimentation

The results obtained from the computational experimentation are presented in this subsection. Two different sets of instances were considered; the main difference is that one has homogeneous production capacity for the facilities and the other one, contains heterogeneous capacities. The former set of instances was taken from the instances used for the capacitated warehouse location problem contained in the Beasley's OR Library. From that set, we consider the 13 instances with 16 facilities and 50 customers, that is, 16×50. Regarding the last four instances, the fixed costs and capacities for each facility were modified in order to reach different optimal solutions. On the other hand, the latter set was taken from a set of instances for the single source capacitated plant location problem studied in [16]. For our experimentation, ten instances of size 15×30 were considered. Both sets were adapted by adding the customer's preferences with the same procedure that is described in [1].

We decided to test only small size instances due to the limitations of the enumerative algorithm. For example, for the 16×50 instances, there are only 26,333 feasible solutions; but, for a set of dimensions 25×50 there are 33,554,431 feasible solutions. Moreover, the lower level is optimally solved for each of these solutions. This clearly limits the capability of the enumerative algorithm for solving medium or large size instances. Also, this work aims to measure the efficiency of the proposed bounds and for doing that, the optimal solution of the tested instances is required.

The computational experimentation was conducted in a workstation with an Intel (R) Core processor and 32GB of RAM. The code was implemented in C++ language and CPLEX 12.6.1 was used for solving the reformulation that omits the lower level objective function, the relaxed reformulation and the lower level in the enumerative algorithm. It is worthy to emphasize that for both sets of instances, the lower level can be optimally solved despite its NP-Hardness. Tables 1 and 2 show the obtained results for instances with homogeneous and heterogeneous production capacity, respectively. For the enumerative algorithm, the optimal upper-level objective function value and the required time are shown. Then, regarding the proposed bounds, the reached values, the required computational time and the optimality gap, computed as$100\% \times (\text{optimum} - \text{bound})/\text{optimum}$, are presented.

Table 1. Results obtained from the computational experimentation with homogeneous production capacity.

	Enumerativealgorithm		**Classicallowerbounds**			**Relaxedreformulationbounds**		
Instance	**Optimum**	**Time (s)**	**Bound**	**Time (s)**	**Gap (%)**	**Bound**	**Time (s)**	**Gap (%)**
$16 \times 50 - 41$	1,518,736	1,697.72	964,895	1.02	36.47	1512700	340.85	-0.42
$16 \times 50 - 42$	1,640,170	1,072.51	1,016,155	1.11	38.05	1737580	372.03	5.94
$16 \times 50 - 43$	1,599,687	907.21	1,064,678	2.20	33.44	1604880	347.09	0.32
$16 \times 50 - 44$	1,648,478	1,001.80	1,129,726	7.19	31.47	1675090	353.44	1.61
$16 \times 50 - 51$	1,416,440	1,186.43	1,014,038	1.56	28.41	1437680	102.60	1.51
$16 \times 50 - 61$	1,307,693	1,230.22	932,616	0.64	28.68	1289980	30.36	-1.35
$16 \times 50 - 62$	1,305,170	1,288.15	977,799	0.56	25.08	1309990	28.56	0.37
$16 \times 50 - 63$	1,322,223	1,234.80	1,010,808	1.02	23.55	1322760	46.89	0.04
$16 \times 50 - 64$	1,405,077	1,260.28	1,042,331	2.02	25.82	1385040	39.86	-1.43
$16 \times 50 - 71$	1,332,155	1,222.64	940,116	0.28	29.43	1360000	29.58	2.09
$16 \times 50 - 72$	1,347,556	1,126.81	990,299	0.39	26.51	1347560	26.78	0.00

$16 \times 50 - 73$	1,346,647	1,192.97	1,028,140	0.45	23.65	1349660	28.08	0.22
$16 \times 50 - 74$	1,352,116	1,140.99	1,060,130	0.52	21.59	1389990	24.31	2.81

From Table 1, it can be appreciated that –as it is expected- the enumerative algorithm consumes a significant amount of time for solving small-sized instances. The scheme for obtaining the classical lower bounds is very fast. However, the optimality gaps are very large –between 21.5% and 38%-. On the other hand, it can be seen that the obtained bounds from the proposed relaxed reformulation have good quality. The greater magnitude of the optimality gap is 5.94%. Also, the time reduction is significant. On the other hand, there are negative and positive optimality gaps, which means that obtaining feasible bilevel solutions for this problem is not guaranteed due to the lower level's relaxation (gaps with negative values are semi-feasible bilevel solutions). Nevertheless, the proposed bounds are a viable option for solving this problem.

Table 2. Results obtained from the computational experimentation with heterogeneous production capacity.

	Enumerativealgorithm		**Classicallowerbounds**			**Relaxedreformulationbounds**		
Instance	**Optimum**	**Time (s)**	**Bound**	**Time (s)**	**Gap (%)**	**Bound**	**Time (s)**	**Gap (%)**
$15 \times 30 - p7$	5,102	5,705.18	4,366	8.14	14.43	5,231.34	5.50	-2.54
$15 \times 30 - p8$	8,908	5,022.95	7,926	7.50	11.02	8,657.60	3.84	2.81
$15 \times 30 - p9$	3,363	2,235.52	2,480	3.33	26.26	3,467.96	10.17	-3.12
$15 \times 30 - p10$	24,012	3,076.26	23,144	7.63	3.61	24,143.56	0.56	-0.55
$15 \times 30 - p12$	4,417	6,186.57	3,711	11.14	15.98	4,459.71	18.83	-0.97
$15 \times 30 - p13$	4,569	4,837.69	3,760	9.67	17.71	4,690.32	21.03	-2.66
$15 \times 30 - p14$	6,700	6,026.05	5,965	6.92	10.97	6,798.35	0.91	-1.47
$15 \times 30 - p15$	8,909	1,079.64	7,816	5.94	12.27	8,728.28	1.77	2.03
$15 \times 30 - p16$	12,262	1,024.27	11,543	6.39	5.86	12,325.25	1.23	-0.52
$15 \times 30 - p17$	10,700	6,299.82	9,884	5.97	7.63	10,673.03	0.66	0.25

Furthermore, when considering instances with a different structure, the behavior of the proposed bounds remains very similar -see Table 2-. In other words, the classical lower bounds are not as strengthened as the ones obtained by the relaxed reformulation. Also, there are some instances which required less time for obtaining the proposed bounds than for obtaining the classical ones. In Figure 1, the optimality gaps associated with the classical lower bounds and the proposed bounds are displayed. The first 13 instances correspond to Table 1 and the remaining ten instances to Table 2. It can be seen that with the reformulation that leads us to lower bounds, larger gaps are obtained than those obtained with the primal-dual relaxed reformulation. For example, the worst gap of the classical lower bounds is 38% and the worst proposed bound is almost 6%. Both bounds correspond to instance$16 \times 50 - 42$. Also, it can be observed that instance $15 \times 30 - p9$ shows the larger negative gap between both sets for the proposed bounds; and the larger gap for the classical bounds in the instances with heterogeneous production capacity.

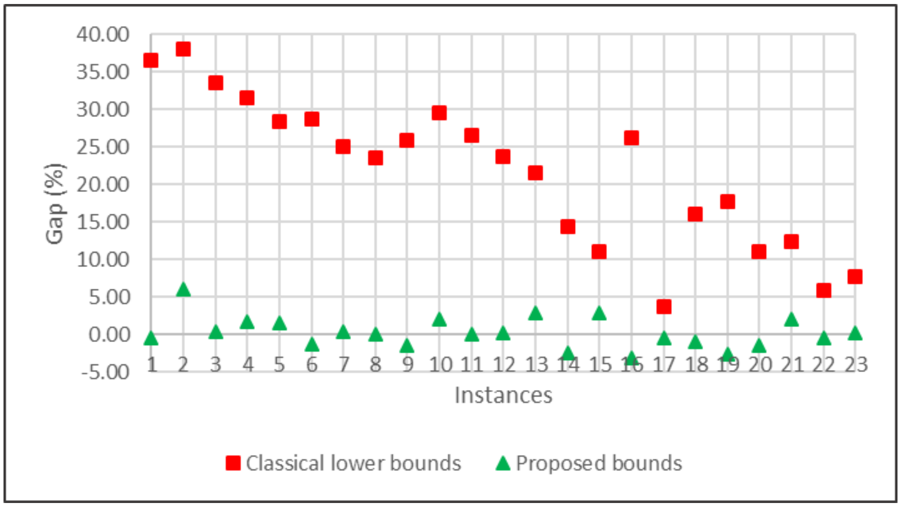

Fig. 1. Optimality gaps.

Figure 2 plots –in a logarithmic scale- the required time for the three considered schemes. The findings are very intuitive, that is, the enumerative algorithm consumes more time than the other two methods. Then, the first reformulation, which gives poor quality bounds, is the less time consuming for the first set of instances. However, for the second set, the proposed relaxed reformulation requires less time in 7 of 10 instances.

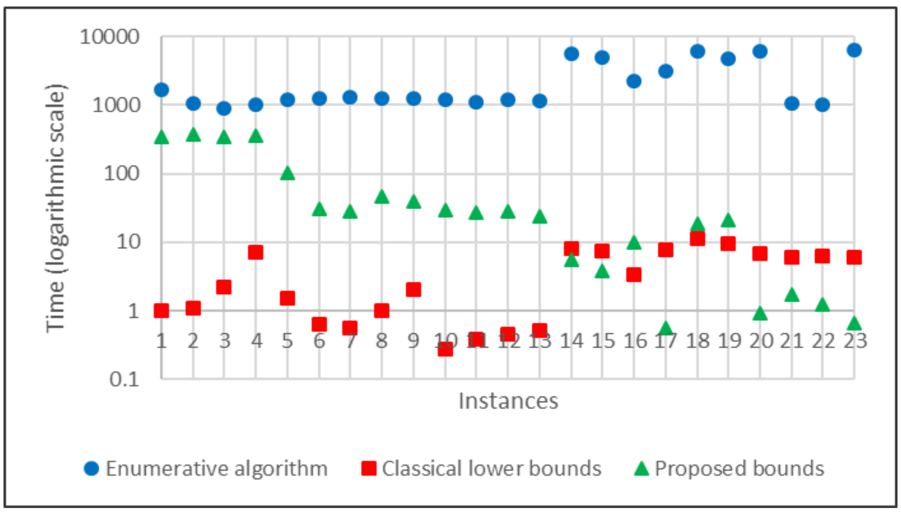

Fig. 2. Consumed time (in seconds).

6. Conclusions

In this paper, a capacitated bilevel facility location problem with customer's preferences is studied. Also, valid bounds for this complex problem were proposed. The main motivation is that the lower level problem is NP-hard, so, the inducible region of the bilevel problem may not be explicitly found. However, some appropriate resolution scheme must be conducted. Hence, in order to approximate the inducible region, a bound based on the linear relaxation of the lower level is proposed. This could lead us to guess that an upper bound will be found. Nevertheless, the computational experience shows that this is not always the case. Therefore, the proposed bound is valid, but it cannot be classified as upper or lower bound.

Moreover, the results obtained from the computational experimentation show that the classical reduction of bilevel programming problems gives poor quality lower bounds. It is well-known that this behavior takes place because the objective function of the lower level is not being taken into consideration during the decision process. In order to compare the obtained bounds, an enumerative algorithm was implemented.

The proposed relaxed reformulation's performance is quite acceptable due to the fact that the required computational time is reasonable –the worst time is only 372 seconds- with good quality bounds for the set of instances with homogeneous production capacity. On the other hand, in the second set of instances, the computational time is decreased in most of the tested instances. Moreover, the good quality of the obtained bounds is maintained. Hence, the proposed bounds seems to be a good option for solving this problem regardless of the structure of the instance.

Two straightforward future research areas can be mentioned. The first one is to experiment with a larger set of instances to validate the well-behavior of the proposed bounds. Based on the inefficiency of the enumerative algorithm when solving medium or large size instances, the second one is to implement efficient exact procedures based on adding valid inequalities for solving the bilevel version of the problem.

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